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Exposé V: Semi-stable reduction and crystalline cohomology with logarithmic poles. (English) Zbl 0852.14004

Fontaine, Jean-Marc (ed.), Périodes p-adiques. Séminaire de Bures-sur-Yvette, France, 1988. Paris: Société Mathématique de France, Astérisque. 223, 221-268 (1994).

We say a scheme X over a discrete valuation ring A is with semi-stable reduction if étale locally on X, there is a smooth morphism $X \to \operatorname{Spec}(A[T_1, \dots, T_r]/(T_1 \cdots T_r - \pi)$ for some $r \geq 0$, where π is a uniformizing parameter. This condition is equivalent to the condition that X is regular, the generic fiber of X is smooth, and the closed fiber of X is a reduced divisor with normal crossings on X. Let A be a complete discrete valuation ring with field of fractions K and with residue field k such that $\operatorname{char}(K) = 0$, $\operatorname{char}(k) = p > 0$, and k is perfect, and let K_0 be the field of fractions of the ring $W = W(k)$ of Witt vectors. Let X be a proper scheme over A with semi-stable reduction, and let $Y = X \otimes_A k$. Then, the crystalline cohomology group $H^m_{\text{crys}}(Y/W) \otimes_W K_0$ $(m \in \mathbb{Z})$ is not a "good cohomology" when Y is singular. However *U. Jannsen* conjectured [in: Galois groups over $\mathbb{Q}$, Proc. Workshop, Berkeley 1987, Publ., Math. Sci. Res. Inst. 16, 315-359 (1989; Zbl 0703.14010)] that there is a "new crystalline cohomology group" D, which is a finite-dimensional K_0-vector space endowed with

– a bijection frobenius-linear operator $\varphi : D \to D$ called the frobenius,

– a nilpotent operator $\mathcal{N} : D \to D$ called the monodromy operator, satisfying $\mathcal{N}\varphi = p\varphi\mathcal{N}$,

– a K-isomorphism with the de Rham cohomology $\rho : D \otimes_{K_0} K \tilde{\to} H^m_{DR}(X_K/K)$ $(X_K = X \otimes_A K)$.

This space D is a mixed characteristic analogue of the limit Hodge structure. The triple $(D, \varphi, \mathcal{N})$ is constructed by *O. Hyodo* [Compos. Math. 78, No. 3, 241-260 (1991; Zbl 0742.14015)] by using some de Rham-Witt complex with logarithmic poles. In this paper, we give another construction of $(D, \varphi, \mathcal{N})$ using the crystalline cohomology theory with logarithmic poles and give the isomorphism ρ. The 4-ple $(D, \varphi, \mathcal{N}, \rho)$ has the following further properties:

– $(D, \varphi, \mathcal{N})$ depends only on the scheme $X \otimes_A A/m_A^2$ over A/m_A^2 where m_A denotes the maximal ideal of A.

– The isomorphism ρ depends on a choice of a prime element π of A.

If we indicate the choice of π as ρ_π, we have $\rho_{\pi u} = \rho_\pi \circ \exp(\log(u)\mathcal{N})$ for $u \in A^\times$, where we denote the K-linear operator on $D \otimes_{K_0} K$ induced by $\mathcal{N}$ by the same letter $\mathcal{N}$. The K-linear operator $\rho_\pi \circ \mathcal{N} \circ \rho_\pi^{-1}$ on $H^m_{DR}(X_K/K)$ is independent of the choice of π. As is shown by *O. Hyodo* [loc. cit.], the triple $(D, \varphi, \mathcal{N})$ is $\otimes_W K_0$ of a triple $(H, \varphi, \mathcal{N})$ with H a canonical defined $W(k)$-module of finite type. *L. Illusie* has proposed a method to show that the operator $\mathcal{N} : H \to H$ is already nilpotent before $\otimes_W K_0$.

The theory of crystalline cohomology with logarithmic poles used in this paper is based on the theory of "logarithmic structures" of Fontaine-Illusie reported by *K. Kato* [in: Périodes p-adiques. Sém. Bures-sur-Yvette 1988, Astérisque 223, 269-293 (1994; Zbl 0847.14009)]. In fact, by using this theory of logarithmic structures, we construct $(D, \varphi, \mathcal{N}, \rho)$ in this paper not only for X as above, but also for a scheme over A with a "smooth logarithmic structure whose reduction is of Cartier type" (for example, a product of schemes with semi-stable reduction is such a scheme). We give also the detailed study of the de Rham-Witt complexes with logarithmic poles associated to such general situation.

The subject of this paper is studied independently by *G. Faltings* [in: The Grothendieck Festschrift, Vol. II, Prog. Math. 87, 219-248 (1990; Zbl 0736.14004)].

For the entire collection see [Zbl 0802.00019].

MSC:

14F30	p-adic cohomology, crystalline cohomology
14L30	Group actions on varieties or schemes (quotients)

Keywords:

logarithmic structures; semi-stable reduction; Witt vectors; crystalline cohomology group; Frobenius; monodromy operator; limit Hodge structure