

Barré-Sirieix, Katia; Diaz, Guy; Gramain, François; Philibert, Georges

A proof of the Mahler-Manin conjecture. (Une preuve de la conjecture de Mahler-Manin.)

(French) [Zbl 0853.11059](#)

Invent. Math. 124, No. 1-3, 1-9 (1996).

This paper answers the question of the transcendence values for the Fourier expansion at infinity of the modular invariant j at algebraic points. More precisely, $j(q)$ is proved to be transcendental over the field of rationals (respectively over the field of p -adic rationals) for any non-zero algebraic point q of the unit disk of the complex plane (respectively of the completion of the field of p -adic rationals). This result conjectured by Mahler in the complex case and by Manin in the p -adic case has numerous applications in the theory of elliptic curves and p -adic L functions. The proof inspired by Mahler's method is based on sharp estimates of the coefficients of the modular polynomials. Let us note that a characteristic p analogue of this result has been proved by *J. F. Voloch* [*J. Number Theory* 58, 55-59 (1996; [Zbl 0853.11061](#))], see also *D. S. Thakur's* proof based on the theory of automata [*J. Number Theory* 58, 60-63 (1996; [Zbl 0853.11060](#))].

Reviewer: [V.Berthé \(Marseille\)](#)

MSC:

[11J91](#) Transcendence theory of other special functions

[11G40](#) L -functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture

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