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The first digit problem and scale-invariance. (English) [Zbl 0853.60002](#)

Marcellini, Paolo (ed.) et al., Partial differential equations and applications. Collected papers in honor of Carlo Pucci on the occasion of his 70th birthday. New York, NY: Marcel Dekker. Lect. Notes Pure Appl. Math. 177, 329-340 (1996).

The author's purpose is to give a countably-additive measure-theoretic derivative of Benford's law

$$\text{Prob}[\text{first significant digit}(\text{base } 10) = k] = \log_{10}[(k+1)/k], \quad k = 1, 2, \dots, 9,$$

[cf. *F. Benford*, Proc. Am. Philos. Soc. 78, 551-572 (1938; [Zbl 0018.26502](#)); *R. A. Raimi*, Am. Math. Monthly 83, 521-538 (1976; [Zbl 0349.60014](#))] based on the classical scale-invariance hypothesis. By appropriate choice of measure space, the restriction to Borel subsets of $[1, 10)$, and use of arguments similar to the reviewer's ones [Proc. Am. Math. Soc. 123, No. 3, 887-895 (1995; [Zbl 0813.60002](#))], it is shown that scale-invariance of the probability measure implies absolute continuity, and Benford's law then follows from the explicit expression of the distribution function and the resulting density.

For the entire collection see [[Zbl 0834.00042](#)].

Reviewer: [T.P.Hill \(Atlanta\)](#)

MSC:

[60A10](#) Probabilistic measure theory

Keywords:

[first digit problem](#); [scale-invariance](#); [significant-digit law](#); [Benford's law](#)