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On the existence of positive solutions of nonlinear second order differential equations.

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The authors establish the following criterion for the existence of a positive solution to the boundary value problem (1) $u''(t) + f(t, u(t)) = 0$, $0 < t < 1$, $a \cdot u(0) - b \cdot u'(0) = 0$, $c \cdot u(1) + d \cdot u'(1) = 0$.

Theorem. Suppose that $f \in C([0, 1] \times [0, \infty); [0, \infty))$; $a, b, c, d \geq 0$; $cb + ac + ab > 0$; $0 < M := \min\{\frac{c+4d}{4(c+d)}, \frac{a+4b}{4(a+b)}\} < 1$ and assume that there exist two distinct positive constants λ, η such that

$$f(t, u) \leq \lambda \left(\int_0^1 k(s, s) ds \right)^{-1} \quad \text{on } [0, 1] \times [0, \lambda],$$
$$\text{and } f(t, u) \geq \eta \left(\int_{\frac{1}{4}}^{\frac{3}{4}} k(\frac{1}{2}, s) ds \right)^{-1} \quad \text{on } [\frac{1}{4}, \frac{3}{4}] \times [M\eta, \eta],$$

where $k(t, s)$ is the Green's function of the differential equation $u''(t) = 0$, $t \in (0, 1)$ with respect to the same boundary conditions as (1). Then (1) has at least one positive solution u such that $|u| := \sup_{t \in [0, 1]} |u(t)|$ between λ and η .

The authors apply this main result to establish several existence theorems of multiple positive solutions for some nonlinear (elliptic) differential equations.

Reviewer: A.Burmistrova (Chelyabinsk)

MSC:

[34B15](#) Nonlinear boundary value problems for ordinary differential equations Cited in **64** Documents

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