The first-order language with = (for sets) under consideration has the constant ∅, the usual binary predicate ∈, and a binary operation symbol w. The intended interpretation of w is: w applied to sets y and z yields y ∪ {z}. NW is the theory axiomatized by (N) ∀x(x ∈ ∅) and (W) ∀x∀y∀z(x ∈ w(y, z) ↔ x ∈ y ∨ x = z). It is well known that Robinson’s Arithmetic, Q, is interpretable in NW + Extensionality. Recently Mancini and Montagna have shown Q is interpretable in NW, showing NW is essentially undecidable.

This paper finds “undecidability” arising low down in the quantificational prefix hierarchy. φ ranges over collections of conjunctions of equalities and one inequality in our given language. The authors prove that the satisfiability of sentences of the form ∀φ with respect to NW is undecidable. They also show that the satisfiability of sentences of the form ∃∀φ with respect to NW + two particular equational consequences of Extensionality for w is also undecidable. The satisfiability of universal sentences is decidable in this extension of NW. The undecidability results derive from a sufficient condition for establishing undecidability for an equational language. It involves coding Turing Machine computations.

For the entire collection see [Zbl 0846.00022].

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MSC:
03B25 Decidability of theories and sets of sentences
03E30 Axiomatics of classical set theory and its fragments

Keywords:
weak membership; decidability; extensionality; coding Turing machine computations; satisfiability of sentences; undecidability; equational language