The first-order language with \( = \) (for sets) under consideration has the constant \( \emptyset \), the usual binary predicate \( \in \), and a binary operation symbol \( w \). The intended interpretation of \( w \) is: \( w \) applied to sets \( y \) and \( z \) yields \( y \cup \{ z \} \). NW is the theory axiomatized by \((N) \ \forall x (x \notin \emptyset)\) and \((W) \ \forall x \forall y \forall z (x \in w(y, z) \leftrightarrow x \in y \lor x = z)\). It is well known that Robinson’s Arithmetic, \( Q \), is interpretable in NW + Extensionality. Recently Mancini and Montagna have shown \( Q \) is interpretable in NW, showing NW is essentially undecidable.

This paper finds “undecidability” arising low down in the quantificational prefix hierarchy. \( \varphi \) ranges over collections of conjunctions of equalities and one inequality in our given language. The authors prove that the satisfiability of sentences of the form \( \forall \varphi \) with respect to NW is undecidable. They also show that the satisfiability of sentences of the form \( \exists \forall \varphi \) with respect to NW + two particular equational consequences of Extensionality for \( w \) is also undecidable. The satisfiability of universal sentences is decidable in this extension of NW. The undecidability results derive from a sufficient condition for establishing undecidability for an equational language. It involves coding Turing Machine computations.

For the entire collection see [Zbl 0846.00022].

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MSC:

- 03B25 Decidability of theories and sets of sentences
- 03E30 Axiomatics of classical set theory and its fragments

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