Ebmeyer, C.

A non-degeneracy property for a class of degenerate parabolic equations. (English)

Zbl 0858.35069

Z. Anal. Anwend. 15, No. 3, 637-650 (1996).

Summary: We deal with the initial and boundary value problem for the degenerate parabolic equation $u_t = \Delta\beta(u)$ in the cylinder $\Omega \times (0,T)$, where $\Omega \subset \mathbb{R}^n$ is bounded, $\beta(0) = \beta'(0) = 0$, and $\beta' \geq 0$ (e.g., $\beta(u) = u|u|^{m-1}$ $(m > 1)$). We study the appearance of the free boundary, and prove under certain hypothesis on β that the free boundary has a finite speed of propagation, and is Hölder continuous. Further, we estimate the Lebesgue measure of the set where $u > 0$ is small and obtain the non-degeneracy property $|\{0 < \beta'(u(x,t)) < \varepsilon\}| \leq c\varepsilon^{1/2}$.

MSC:

35K65 Degenerate parabolic equations
35R35 Free boundary problems for PDEs
76S05 Flows in porous media; filtration; seepage

Keywords:

speed of propagation; porous medium equations

Full Text: DOI

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