Meester, Ronald; Roy, Rahul
Continuum percolation. (English) Zbl 0858.60092
Cambridge Tracts in Mathematics. 119. Cambridge: Cambridge Univ. Press. x, 238 p. (1996).

The book is the first systematic and rigorous treatment of continuum percolation. In discrete bond percolation one considers e.g. the square lattice Z^d. Here, the set of vertices is Z^d, and the possible edges are those between vertices at Euclidean distance one apart. Let $0 \leq p \leq 1$. A certain edge occurs with probability p and does not occur with probability $1-p$ (different edges behaving independently). This yields a random graph with a deterministic (!) set ($= Z^d$) of vertices and a random set of edges. Let $\mathrm{card}(C(0))$ denote the cardinality of the set of vertices belonging to the connected component $C(0)$ which contains the origin $0 \in Z^d$. By definition, the percolation function $\theta^{(d)}(p)$ equals the probability that $\mathrm{card}(C(0)) = \infty$. The corresponding critical probability is defined by $p_c(d) = \inf\{p : \theta^{(d)}(p) > 0\}$. Interesting problems include e.g.: What is the value of $p_c(d)$? Is there only one infinite connected component? How does the model behave in the critical case where $p = p_c(d)$?

One arrives at continuum percolation if also the set of vertices is taken to be random. In the present book, the vertex set is given by some stationary point process X on R^d. In Chapters 3-6, X is a Poisson process. There are two basic models of continuum percolation which are mainly studied: The Boolean model and the random-connection model (RCM). In the first of these, each point of X is the centre of a closed ball (in the usual Euclidean metric) such that the radii are i.i.d. and are also independent of X. In this case, there is the occupied region which equals the union of the balls, and its complement which is called the vacant region. Let W (corresponding to $C(0)$?) denote the connected component of the occupied region, containing the origin. Similarly, V denotes the corresponding component of the vacant region. In the RCM a non-increasing function $g : R_+ \to [0,1]$ is given such that any two points x_1, x_2 of X are connected by an edge with probability $g(|x_1 - x_2|)$ independently of all other pairs of points ($|\cdot|$ denoting the Euclidean norm). In the resulting random graph let W denote the connected component containing the origin 0 – given X has a point at 0. For these – and related – models one can define certain critical probabilities and one can study certain properties of W and V analogous to those of $C(0)$ in discrete bond percolation.

In an introductory chapter, it is shown e.g. how the Boolean model and the RCM can be rigorously constructed. Chapter 2 presents some basic results and tools (ergodicity, FKG and BK inequalities, coupling and scaling). In Chapters 3-6 basic properties of the Poisson Boolean model and the Poisson RCM are studied (this includes e.g. critical phenomena, critical densities, exponential decay of W and the RSW lemma being an analogue of the RSW lemma of discrete percolation due to *L. Russo* [Z. Wahrscheinlichkeitstheorie Verw. Geb. 43, 39-48 (1978; Zbl 0363.60120)] and *D. D. Seymour* and *D. J. A. Welsh* [Ann. Discrete Math. 3, 227-245 (1978; Zbl 0405.60015)]). In Chapter 7, models driven by more general stationary point processes are studied. Finally, Chapter 8 is dealing with related models including continuum fractal percolation, and percolation of level sets in random fields.

The book is friendly and carefully written. All important techniques and methods are explained and motivated. The authors tried hard to make the book as self-contained as possible. According to the opinion of the reviewer it is ideally suited for self-study.

Reviewer: K.Schürger (Bonn)

MSC:

60K35	Interacting random processes; statistical mechanics type models; percolation theory
60-02	Research exposition (monographs, survey articles) pertaining to probability theory
60D05	Geometric probability and stochastic geometry

Keywords:

continuum percolation; random graph; stationary point process; Boolean model; random-connection model; FKG and BK inequalities; continuum fractal percolation

Full Text: DOI