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Naturally reductive Riemannian homogeneous structures on some classes of generic submanifolds in complex space forms. (English) Zbl 0860.53032

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Let $M^n(c)$ be a non-flat complex space form. Geodesic hyperspheres, horospheres and tubes about totally geodesic $M^n(c)$ are equipped with a naturally reductive homogeneous structure and hence, are naturally reductive homogeneous spaces in the simply connected case.

In the present paper, the author provides new examples of submanifolds of $M^n(c)$ equipped with such a structure which are not (locally) symmetric. He proceeds as follows: First, let $c > 0$. Any Riemannian product of a finite number of odd-dimensional spheres can be embedded in an odd-dimensional sphere of an appropriate radius. This projects via the Hopf map onto a generic submanifold of a complex projective space. In this case the author gives the explicit expression of a naturally reductive structure on these submanifolds. Further, similar examples are constructed for the case $c < 0$ by using the corresponding Hopf map $\pi : H_1^{2n+1}(r) \to \mathbb{C}H_n$ where $H_1^{2n+1}(r)$ denotes the anti-de Sitter space of radius r, and by considering products of the form $S^{2m_k+1}(r_k) \times \cdots \times S^{2m_1+1}(r_1) \times H_1^{2m_0+1}(r_0)$.

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MSC:

53C40 Global submanifolds
53C30 Differential geometry of homogeneous manifolds
53B20 Local Riemannian geometry
53C55 Global differential geometry of Hermitian and Kählerian manifolds

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