

**Holden, Helge; Øksendal, Bernt; Ubøe, Jan; Zhang, Tusheng**

**Stochastic partial differential equations. A modeling, white noise functional approach.**

(English) [Zbl 0860.60045](#)

Probability and Its Applications. Basel: Birkhäuser. 230 p. (1996).

The authors give a comprehensive introduction to stochastic partial differential equations. Their approach is based on white noise analysis. In Chapter 2 some of the mathematical background is discussed. The authors choose the following approach: rather than considering distribution-valued stochastic processes  $\omega \rightarrow u(\cdot, \omega) \in H^\alpha(\mathbb{R}^d)$ ,  $\omega \in \Omega$ , where  $H^\alpha$  is a Sobolev space, they consider functions  $x \rightarrow u(x, \cdot) \in (S)_{-1}$ ,  $x \in \mathbb{R}^d$ , where  $(S)_{-1}$  is a space of stochastic distributions (called the Kondratiev space). The Wiener-Itô chaos expansion by Hermite polynomials and by multiple Itô integrals is introduced. The properties of the Wick product and the Hermite transform are proved. In Chapter 3 the framework that the authors developed in Chapter 2 is used to obtain new results, as well as new proofs of old results, for stochastic ordinary differential equations. Existence and uniqueness theorems for linear stochastic differential equations, as well as for stochastic linear Volterra equations are proved. Also the Wick approximation of quasilinear SDE is given. In Chapter 4 the authors apply the general theory developed in Chapter 2 to solve various stochastic partial differential equations. First the stochastic Poisson equation and the stochastic transport equation are discussed. Then the authors consider the stochastic Schrödinger equation, as well as the stochastic heat equation. The nonlinear Burger's equation with a stochastic source is discussed, and finally the stochastic pressure equation is treated. The white noise approach often allows for solutions given by explicit formulas in terms of expectations of certain auxiliary processes. A collection of exercises at the end of each chapter is presented.

Reviewer: [A.D.Borisenko \(Kiev\)](#)

**MSC:**

- 60H15** Stochastic partial differential equations (aspects of stochastic analysis)
- 60-02** Research exposition (monographs, survey articles) pertaining to probability theory

Cited in **7** Reviews  
Cited in **115** Documents

**Keywords:**

introduction to stochastic partial differential equations; white noise analysis; Sobolev space; Wiener-Itô chaos expansion; Hermite polynomials; stochastic linear Volterra equations; nonlinear Burger's equation