Ryan, Charles T.

We investigate the geometry of linear dependences associated to the Grassman Variety $G(2,n)$. Linear dependences of order three correspond by Tallini’s Theorem to linear pencils of $G(2,n)$ and an understanding of this geometry enables us to enumerate them. Linear dependences of order four also correspond to a unique configuration, and a further understanding of the geometry of $G(2,n)$ will enable their enumeration as well. Finally we consider linear dependences of order five for the particular Grassmann Variety $G(2,4)$ where it will be shown that the corresponding configuration is either a spread of spaces or arises by replacing a space of a configuration corresponding to an order four linear dependency with a linear pencil of $G(2,n)$. In the first case our enumeration depends on the well known fact that there are exactly 56 spreads of $G(2,4)$ and we give an elementary proof based on the action of the General Linear group $Gl(4)$. The remaining enumeration is then accomplished with the help of the MacWilliams duality formula applied to a code whose generator matrix has as its columns the images of the points of $G(2,4)$ under the classical Plücker embedding.

MSC:
51M35 Synthetic treatment of fundamental manifolds in projective geometries (Grassmannians, Veronesians and their generalizations)
14M15 Grassmannians, Schubert varieties, flag manifolds
11T71 Algebraic coding theory; cryptography (number-theoretic aspects)

Keywords:
linear dependence; algebraic coding theory; Grassmann variety $G(2,n)$