
The operation of producing the successor ordered 5-arc is reversible, the reverse operation being to produce from a given ordered 5-arc \( A_0, \ldots, A_4 \), its predecessor ordered 5-arc whose vertices \( A_{-i} \) are defined by

\[
A_{-i} = A_i + 1 A_i + 2 \cap A_i - 1 A_i - 2.
\]

In this paper we ask as many questions as we answer. Here are some examples. We can show that the successor and predecessor operations described above do in fact yield ordered 5-arcs from any 5-arc in any projective plane of order at least 4. From the fact that these operations are one-to-one mappings on the collection of ordered 5-arcs, it follows that when we iterate our motion both ways we form a double-ended sequence of 5-arcs or else a cycle of them having no tail. This partitions all the ordered 5-arcs of any projective plane. Moreover, we can find infinite planes of the following three types: (i) there are only cycles, (ii) there are no cycles, and (iii) there are both cycles and double-ended sequences. Our first unanswered question is: given certain infinite planes, such as the Euclidean plane, which of (i), (ii), or (iii) occur? We will relate this to the Sylvester-Gallai problem.

Further examples are given.

**MSC:**

- 51A05 General theory of linear incidence geometry and projective geometries
- 51E21 Blocking sets, ovals, \( k \)-arcs
- 51A35 Non-Desarguesian affine and projective planes
- 51E15 Finite affine and projective planes (geometric aspects)

**Keywords:**

ordered 5-arcs; projective plane; successor; predecessor