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Ergodic theorems along sequences and Hardy fields. (English) Zbl 0863.28011

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Summary: Let $a(x)$ be a real function with a regular growth as $x \to \infty$. [The precise technical assumption is that $a(x)$ belongs to a Hardy field.] We establish sufficient growth conditions on $a(x)$ so that the sequence $([a(n)])_{n=1}^{\infty}$ is a good averaging sequence in L^2 for the pointwise ergodic theorem. A sequence (a_n) of positive integers is a good averaging sequence in L^2 for the pointwise ergodic theorem if in any dynamical system (Ω, Σ, m, T) for $f \in L^2(\Omega)$ the averages

$$\frac{1}{X} \sum_{n \leq X} f(T^{a_n} \omega)$$

converge for almost every $\omega \in \Omega$. Our result implies that sequences like $([n^{\delta}])$, where $\delta > 1$ and not an integer, $([n \log n])$ and $([n^2 / \log n])$ are good averaging sequences for L^2. In fact, all the sequences we examine will turn out to be good averaging for L^p, $p > 1$; and even for $L \log L$.

We also establish necessary and sufficient growth conditions on $a(x)$ so that the sequence $([a(n)])$ is good averaging for mean convergence. Note that for some $a(x)$ (e.g., $a(x) = \log^2 x$), $([a(n)])$ may be good for mean convergence without being good for pointwise convergence.

MSC:

28D05 Measure-preserving transformations

26A12 Rate of growth of functions, orders of infinity, slowly varying functions

Keywords:

Hardy fields; growth conditions; good averaging sequence; pointwise ergodic theorem; averages; mean convergence; pointwise convergence

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