Ito, Noboru

Some results on Hadamard groups. (English) Zbl 0864.05021

Kim, A. C. (ed.) et al., Groups - Korea '94. Proceedings of the international conference, Pusan, Korea, August 18-25, 1994. Berlin: Walter de Gruyter. 149-155 (1995).

Let G be a group of order $2n$ such that G contains a central involution e^*. Let D be a transversal of G with respect to subgroup N generated by e^*. Then there exist exactly 2^n transversals. For every transversal D we have that $D = De$, where e denotes the identity element of G, $D \cap De^* = \varnothing$ and $G = D \cup De^*$. If for some transversal D we have that $|D \cap Da| = n/2$ for every element a of G other than e and e^*, where $|X|$ denotes the number of elements in a finite set X, namely if D is a $(n, n/2, N)$ relative difference set, then we call D and G an Hadamard subset and an Hadamard group respectively. Hadamard groups are introduced so that we may scrutinize the structure of automorphism groups of Hadamard matrices.

Let D be any transversal. Then it holds obviously that

$$|D \cap Da| = |D \cap Da^{-1}| \quad \text{for any element } a \text{ of } G \tag{1}$$

and that

$$|D \cap Da| + |De^* \cap Da| = n \quad \text{for any element } a \text{ of } G. \tag{2}$$

From (1) and (2) follows the following lemma.

Lemma 1. Let D be any transversal. If a is an element of order 4 such that $a^2 = e^*$, then we have that $|D \cap Da| = n/2$.

Lemma 2. Let H be a non-trivial subgroup of G. If a transversal D is a union of left cosets of H, then D is not an Hadamard subset.

Proposition 1. Let C_m and Q denote the cyclic group of order m and the quaternion group of order 8 respectively. Let G be an Hadamard group with the property that every transversal is an Hadamard subset. Then G is isomorphic to either C_4 or Q.

Proposition 2. Let G be an Hadamard group of order $8n$ and S a Sylow 2-subgroup of G. Assume that $S = C_{2^{m+1}} \times C_2$ and that a and b are generators for $C_{2^{m+1}}$ and C_2 respectively. If $e^* = a^{2^m}$, then $m = 1$ or 2.

(Remarks on the strong Hadamard conjecture and a conjecture of Ryser follow).

For the entire collection see [Zbl 0857.00028].

MSC:

05B20	Combinatorial aspects of matrices (incidence, Hadamard, etc.)	Cited in **2** Reviews
20D60	Arithmetic and combinatorial problems involving abstract finite groups	Cited in **3** Documents

Keywords:

transversal; difference set; Hadamard subset; Hadamard group; Hadamard matrices