Jimbo, Michio; Kedem, Rinat; Konno, Hitoshi; Miwa, Tetsuji; Petersen, Jens-Ulrik H.
Level-0 structure of level-1 $U_q(\hat{sl}_2)$-modules and Macdonald polynomials. (English)

The authors study a curious finding in representation theory, noted recently in connection with conformal field theory (CFT) solvable lattice models. In the context of quantum affine algebras, it can be stated as follows: Let $V(\Lambda_i) (i = 0, 1)$ be a level-1 integrable highest weight module of $U_q(\hat{sl}_2)$. Then it admits a level-0 action of the same algebra $U_q(\hat{sl}_2)$. This second action leaves invariant each homogeneous component $V(\Lambda_i)_{-n}$, and the whole module becomes a direct sum of finite-dimensional irreducible constituents, which can be described explicitly.

This observation traces back to the study of spin chains with long-range interactions (the Haldane-Shastry model) [F. Haldane et al., Phys. Rev. Lett. 69, 2021-2025 (1992)]. This model has the remarkable property that the Yangian $Y(\hat{sl}_n)$ acts as an exact symmetry even for finite chains. By considering the continuum limit, it was then suggested, and subsequently confirmed, that there is an action of the Yangian on level-1 integrable modules of the affine Lie algebra $\hat{sl}_n$. This action is related to the fermionic character formula for the level-1 module conjectured by E. Melzer [Lett. Math. Phys. 31, 233-246 (1994; Zbl 0797.17018)]. Similar results are expected to hold for higher level representations as well. The fermionic expressions conjectured there for the characters of the level-$k$ modules, subsequently proved, strongly support the validity of this picture.

Here the authors discuss a $q$-deformation of these structures in the simplest case of $U_q(\hat{sl}_2)$ with level 1. Namely they consider level-1 modules of $U_q(\hat{sl}_2)$, and define a level-0 action on them. This action is defined using the affine Hecke algebra and the basis of the level-1 module generated by components of vertex operators. Each level-1 module is a direct sum of finite-dimensional irreducible level-0 modules, whose highest weight vector is expressed in terms of Macdonald polynomials. This decomposition leads to the fermionic character formula for the level-1 modules. The quantum affine algebra with level 0 plays the role of the Yangian in CFT. It is not yet clear whether such a $q$-deformation is related to some physical models like the Haldane-Shastry chain. Apart from the technical complexity, the essence of the construction does not differ very much from the conformal case. Their main aim here is to supply the mathematical details and give a coherent account of this as yet mysterious phenomenon.

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MSC:
17B37 Quantum groups (quantized enveloping algebras) and related deformations
17B67 Kac-Moody (super)algebras; extended affine Lie algebras; toroidal Lie algebras
82B23 Exactly solvable models; Bethe ansatz
81T40 Two-dimensional field theories, conformal field theories, etc. in quantum mechanics

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