Birman, M. Sh.; **Laptev, A.**
The negative discrete spectrum of a two-dimensional Schrödinger operator. (English)
Zbl 0864.35080
Commun. Pure Appl. Math. 49, No. 9, 967-997 (1996).

In the main part of the paper, the non-Weyl term β in the asymptotics for the number of the negative eigenvalues of the Schrödinger operator in $L_2(\mathbb{R}_2)$, $-\Delta-\alpha V$,

$$\lim_{\alpha\to\infty}\frac{\#\{\lambda_s<0\}}{\alpha}=\frac{1}{4\pi}\int V\,ds+\beta$$

is analyzed. Two-sided estimates of β are given in terms of asymptotic behaviour of the eigenvalues of the auxiliary polar problem $-d^2\omega/dt^2=\frac{1}{\lambda}F(t)\omega(t)$, $A<t<\infty$ with some selfadjoint boundary conditions at A,

$$F(t)=e^{2t}\int_0^{2\pi}V_+(e^t,\theta)d\theta.$$

If for some p, $\delta_p/s^p\le\{\#(\lambda_k>s)\}\le\Delta_p/s^p$, then $\delta_1<\beta<\Delta_1$ for $p=1$ or

$$\overline{\lim_{\alpha\to\infty}}\,\alpha^{-p}\{\#(\lambda_k>s)\}=\Delta_p,\qquad \underline{\lim_{\alpha\to\infty}}\,\alpha^{-p}\{\#(\lambda_k>s)\}=\delta_p,$$

for $p>1$. The proof is based on the elegant idea of "splitting" the quadratic form of the Schrödinger operator into two parts, one of them defined on the subspace of the functions not depending on angular variables, and the other involves the orthogonal complement. The first term corresponds to the auxiliary problem, since for $u_0=\frac{1}{2\pi}\int u\,d\theta$,

$$\int_{|x|>a}|u_0|^2V\,dx=\int_{r>a}|u_0(r)|^2\cdot r\int_0^{2\pi}V(r,\theta)d\theta\cdot dr.$$

In the second part of the paper, the multi-dimensional analog of the problem is considered, which corresponds to the Schrödinger operator with the singular potential

$$-\Delta-\frac{(d-2)^2}{|x|^2}-\alpha V,\qquad x\to\infty.$$

The non-Weyl series of the asymptotic formula are analyzed. At the end of the paper a similar result concerning the local singularities of the potential is announced.

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MSC:

35P20 Asymptotic distributions of eigenvalues in context of PDEs
35J10 Schrödinger operator, Schrödinger equation

Keywords:
number of negative eigenvalues; non-Weyl term; singular potential; local singularities

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