Let $Q_n[f] = \sum_{\nu=1}^n a_{\nu} f(x_{\nu})$ be a quadrature formula (q.f.) for $I[f] = \int_{-1}^{1} f(x) dx$ with error $R_n[f] = I[f] - Q_n[f]$. Set $\rho(Q_n, K) = \sup_{f \in K} |R_n[f]|$, then $\rho_n(K) = \inf_{Q_n} \rho(Q_n, K)$ is the error of an optimal q.f. for the class $K$. Here optimal is among all $n$ point q.f. If a q.f. $Q_n$ is used instead of the optimal q.f., one looses a factor $\text{loss}(Q_n, K) = \rho(Q_n, K)/\rho_n(K)$ in accuracy. A Gaussian q.f. $Q_n^G$ is said to be universal (for $K_s$) if $M(n) = \sup_{1 \leq s \leq 2n} \text{loss}(Q_n^G, K_s)$ is “small”. The index $s$ refers to the smoothness of the functions $f$. In this paper

$K_s = \{ f \in C^s[-1,1] : \int_{-1}^{1} [f^{(s)}(x)]^2 (1-x^2)^{s-1/2} dx < 1 \}$

This notion of universal q.f. is an adaptation of a more general approach given by H. Brass, ISNM 85, 16-24 (1988; Zbl 0648.41015). In this paper it is shown that $M(n)$ as defined above is bounded by $(22/3)n^{3/4} + 1$.

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