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A Riemannian invariant for submanifolds in space forms and its applications. (English)
Zbl 0865.53050

Dillen, Franki (ed.) et al., Geometry and topology of submanifolds, VI. Proceedings of the international meetings on 'Pure and applied differential geometry' and on 'Theory of submanifolds' which were held in combination on July 10, 11, 14, 1993 at Leuven and on July 12, 13, 1993 at Brussels, Belgium. Singapore: World Scientific. 58-81 (1994).

Let M^n be an n-dimensional submanifold of a real space form $\widetilde{M}(c)$ with constant sectional curvature c. In another paper [Arch. Math. 60, No. 6, 568-578 (1993; Zbl 0811.53060)], the author proved that the Riemannian invariant δ_M of M^n, defined by $\delta_M(p) = \tau(p) - \inf K(p)$, where $\inf K(p)$ is the smallest sectional curvature at $p \in M^n$ and $\tau = \sum_{i<j} K(e_i \wedge e_j)$ is the scalar curvature, satisfies the following inequality:

$$\delta_M \leq \frac{n^2(n-2)}{2(n-1)}|H|^2 + \frac{1}{2}(n+1)(n-2)c,$$

where H is the mean curvature vector. This inequality is sharp, and many nice classes of submanifolds realize equality in this inequality.

The paper under review surveys a number of applications and a number of classification results for submanifolds realizing the equality. Essentially there are two types of applications discussed. The first type of applications are intrinsic obstructions for Riemannian manifolds to allow a minimal immersion into real space forms. For instance it is proved that there are no minimal immersions of the product of the hyperbolic plane and some Euclidean space into Euclidean space. The second type of applications are estimates for the length of the mean curvature vector. Next, classifications of some classes of submanifolds realizing the equality are given: minimal submanifolds of Euclidean spaces, isoparametric hypersurfaces, tubular hypersurfaces, reducible submanifolds and totally real submanifolds of the nearly Kähler 6-sphere. In the final part, similar results are discussed about totally real submanifolds of complex space forms, for which the same inequality holds.

For the entire collection see [Zbl 0832.00044].

Reviewer: F.Dillen (Leuven)

MSC:

53C42 Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)
53C40 Global submanifolds

Keywords:

Riemannian invariant; mean curvature vector; real space forms; minimal immersions