Summary: Let $R$ be an associative ring and $f$ be an injective endomorphism of $R$ such that the Cohn-Jordan extension $A(R, f)$ satisfies the ascending chain condition on left annihilators. In this paper we obtain some semiprimitivity criteria for the skew polynomial ring $R[x, f]$ over the ring $R$. In particular, we prove that the skew polynomial ring is semisimple if and only if its prime radical is zero. Furthermore, it is so if and only if the ring $R$ is semiprime.

MSC:

16S36 Ordinary and skew polynomial rings and semigroup rings
16D60 Simple and semisimple modules, primitive rings and ideals in associative algebras
16N60 Prime and semiprime associative rings

Keywords:

injective endomorphisms; ascending chain condition on left annihilators; semiprimitivity criteria; skew polynomial rings; prime radical