

Madsen, Ib

The cyclotomic trace in algebraic K -theory. (English) Zbl 0866.55009

Joseph, A. (ed.) et al., First European congress of mathematics (ECM), Paris, France, July 6-10, 1992. Volume II: Invited lectures (Part 2). Basel: Birkhäuser. Prog. Math. 120, 213-241 (1994).

The paper is a concise and informative account of the relation between algebraic K -theory and topological cyclic homology. It is essentially written in an expository manner and can be recommended to anybody who is interested in a quick review of the constructions and the most important results. Of course, like most of the expository paper it has to be read with the appropriate care.

The paper is organized as follows: After the introduction the basic definitions are sketched in §2. Topological Hochschild homology $THH(F)$ for a functor F with smash product (FSP) is defined, its structure as a p -cyclotomic spectrum described and topological cyclic homology at p $TC(F, p)$ is introduced. As a first example the p -completed version $TC(F, p)_p^\wedge$ for $F = \text{Id}$ is discussed. Then the K -theory $K(F)$ of F is defined and the construction of the cyclotomic trace $\text{Trc} : K(F) \rightarrow TC(F, p)$ is sketched. In §3 evidence is given for the following conjecture.

Conjecture: Let A be a ring with ideal I such that $A = \lim A/I^n$, A/I is a finite semi-simple \mathbb{F}_p -algebra, and A is finitely generated as \mathbb{Z}_p -module. Then

- (1) $\text{Trc} : K(A)_p^\wedge \simeq TC(A, p)_p^\wedge$
- (2) $K(A)_p^\wedge \simeq K^c(A)_p^\wedge$ where $K^c(A) = \text{holim } K(A/I^n)$.

In certain cases the conjecture reduces to the following relative version which is known to be true by a recent result of R. McCarthy: Let $R \rightarrow S$ be a surjection of rings with nilpotent kernel then

$$\begin{array}{ccc} K(R)_p^\wedge & \rightarrow & TC(R, p)_p^\wedge \\ \downarrow & & \downarrow \\ K(S)_p^\wedge & \rightarrow & TC(S, p)_p^\wedge \end{array}$$

is homotopy cartesian.

A survey of calculational results for $TC(R, p)_p^\wedge$ follows including the cases $R = \mathbb{F}_p, \mathbb{F}_p[v, v^{-1}], \mathbb{Z}_p$ and the dual numbers $\mathbb{F}_p[\varepsilon]$. Section 4 outlines as an example the calculation of $TC(\mathbb{F}_p, p)_p^\wedge$. The final section discusses the cyclotomic trace for Waldhausen's A -theory and possible applications to homotopy calculations for $\widehat{Top}(M)/Top(M)$, where M is a manifold with negative sectional curvature.

For the entire collection see [Zbl 0807.00008].

Reviewer: **R. Vogt (Osnabrück)**

MSC:

55P42 Stable homotopy theory, spectra
19D10 Algebraic K -theory of spaces

Cited in **1** Document

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topological Hochschild homology; algebraic K -theory; topological cyclic homology; functor; smash product; cyclotomic trace