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On the general linear group over weakly Noetherian associative algebras. (Russian. English summary) [Zbl 0867.20037]

Summary: Let $R$ be a weakly Noetherian algebra with unity element over an infinite field, $I$ an ideal in $R$, $n \geq 3$, $E_n(R)$ the elementary subgroup in the general linear group $GL_n(R)$, $E_n(R, I)$ the normal subgroup in $E_n(R)$ generated by the elementary matrices $1 + \lambda e_{ij}$, $\lambda \in I$, $1 \leq i \neq j \leq n$, $GL_n(R, I)$ the kernel and $C_n(R, I)$ the preimage of the center of the homomorphism $GL_n(R) \to GL_n(R/I)$ respectively. It is proved that if $G$ is a subgroup of $GL_n(R)$, then it is normalized by $E_n(R)$ if and only if $E_n(R, F) \subseteq G \subseteq C_n(R, F)$ for some ideal $F$ of $R$; $[C_n(R, F), E_n(R)] = E_n(R, F)$ and in particular the groups $E_n(R)$ and $E_n(R, F)$ are normal in $GL_n(R)$ for all ideals $F$ of $R$.

MSC:
20G35 Linear algebraic groups over adeles and other rings and schemes
20E07 Subgroup theorems; subgroup growth
20F12 Commutator calculus

Keywords:
weakly Noetherian algebras; elementary subgroups; general linear groups