Bättig, D.; **Kappeler, T.**; **Mityagin, B.**

On the Korteweg-de Vries equation: Convergent Birkhoff normal form. (English)

Zbl 0868.35099
J. Funct. Anal. 140, No. 2, 335-358 (1996).

The authors consider the Korteweg-de Vries (KdV) equation

$$V_t + V_{xxx} - 6V \cdot V_x = 0$$

on the circle, i.e., for functions $V = V(x,t)$ on $S^1 \times \mathbb{R}$. It is well-known that the KdV equation represents a completely integrable Hamiltonian system (of infinite dimension) with phase space $H^n(S^1;\mathbb{R})$, where n is a positive integer, and with a Poisson structure inducing a symplectic structure on the phase space.

The main result of the present paper states that there is a symplectic morphism from the zero-leaf of the phase space, with respect to the Casimir function defined by the Poisson structure to some symplectic Hilbert space. This symplectomorphism is bijective, real-analytic in both directions, and induces globally defined real-analytic action-angle coordinates for the entire KdV hierarchy. This theorem provides the (apparently) first infinite-dimensional counterpart of the result by *J. Vey* [Am. J. Math. 100, 591-614 (1978; Zbl 0384.58012)], which states that a finite-dimensional completely integrable Hamiltonian system with real-analytic Hamiltonian functional admits action-angle variables in a neighborhood of a non-resonant elliptic fixed point.

In the second part of the paper, the authors apply their main result to the study of the regularity of the KdV-Hamiltonian vector field. In particular, the local existence of a convergent Birkhoff normal form is proved.

Reviewer: W.Kleinert (Berlin)

MSC:

35Q53	KdV equations (Korteweg-de Vries equations)
37J35	Completely integrable finite-dimensional Hamiltonian systems, integration methods, integrability tests
37K10	Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)
37J99	Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems

Keywords:

Korteweg-de Vries equation; Hamiltonian systems; Poisson structures; symplectic spaces; Birkhoff normal form

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