Thompson, A. C.

Minkowski geometry. (English) Zbl 0868.52001

Encyclopedia of Mathematics and Its Applications. 63. Cambridge: Cambridge University Press. xiii, 346 p. (1996).

Minkowski geometry here means the geometry of finite dimensional normed spaces. The book starts with fundamental facts: Existence and uniqueness of Haar measures on abelian topological groups, the Hahn-Banach theorem, Löwner ellipsoids, characterization of ellipsoids and Euclidean norms, the Hausdorff metric, the Banach-Mazur distance of normed spaces, and other topics. Here proofs are given whereas in the chapter on convex bodies, mixed volumes, the polar body, and inequalities, the results are presented with at most outlines of proofs. Thereafter two-dimensional Minkowski spaces are studied: the isoperimetric problem, constant width, perimeter of the unit ball, transversality, Radon curves, among others. The central topic of the book is surface area, especially for hypersurfaces, and the isoperimetric problem. This is first treated under general assumptions. Then definite area measures are specified. Among them two have found great interest and are discussed here in detail: 1) the Choquet-Busemann area, 2) the Holmes-Thompson area. In the first case the normalization of $(d-1)$-measure is achieved in the following way: If U is $(d-1)$-dimensional subspace and B is the unit ball, then $U\cap B$ is assigned the volume ε_{d-1} of the Euclidean $(d-1)$-ball. The isoperimetrix, the solution of the isoperimetric problem, is the polar body of the intersection body of B. The Holmes-Thompson area of $U\cap B$ is the product of its volume and the volume of its polar body in U. This less immediate definition has some advantages, for instance in integral geometry. The isoperimetrix now is the projection body of the polar body of B and is always a zonoid. For zonoids the Mahler-Reisner inequality is available, which gives the minimum of the product of the volumes of a body and its polar body. For both area definitions examples of isoperimetrices are determined and illustrated by computer drawings. Estimations for the area of ∂B are derived. Then a chapter on angle measures and trigonometric formulas follows. The last chapter is centered around J. J. Schäffer's work on girth, perimeter and other numerical values associated with the unit ball. Every chapter closes with comprehensive notes on history and literature so that the book is at the same time an introduction, a survey, and a reference manual. At the end there is a collection of fifty problems which appear within the text.

Reviewer: E.Heil (Darmstadt)

MSC:

52-02 Research exposition (monographs, survey articles) pertaining to convex and discrete geometry

52A21 Convexity and finite-dimensional Banach spaces (including special norms, zonoids, etc.) (aspects of convex geometry)

Keywords:

Minkowski geometry; finite dimensional normed spaces; surface area; Choquet-Busemann area; Holmes-Thompson area; isoperimetrix