

**Martinet, Jacques**

**Perfect lattices of Euclidean spaces. (Les réseaux parfaits des espaces euclidiens.)** (English)

Zbl 0869.11056

Paris: Masson. 438 p. (1996).

This book is devoted to a classical subject in the geometry of numbers and recent progress in this area. Its first 150 pages systematically develop the basic geometric properties of lattices in Euclidean spaces, and can serve very well for a course on the whole field. In comparison with the older literature the treatment here is strongly influenced by algebraic thinking. On the other hand the scope is not on integral lattices; therefore and by its style this book differs much from the well-known one by Conway and Sloane.

Chapter 1 introduces the basic vocabulary, Chapter 2 discusses the geometric inequalities and lattice constants of Hermite, Minkowski, Mordell, Rankin and Bergé-Martinet. Chapter 3 gives an elegant account of Voronoi's theorem, characterizing extreme lattices (i.e. those giving a local maximum of the sphere packing density) by the properties of perfection and eutaxy. The next two chapters discuss root lattices and their relatives (Coxeter, Barnes, Leech, Craig). Chapters 6 and 7 expose Voronoi's algorithm for perfect lattices, or rather perfect quadratic forms, the classification of such lattices up to dimension 5, and the corresponding results in dimension 6 (Barnes) and 7 (Stacey, confirmed by Jaquet). The subsequent chapters present more recent developments, for the most part based on work by the author's research group at Bordeaux. These topics are: hermitian structures on lattices, the classification of eutactic lattices (up to dimension 4), a comprehensive notion of extremality for families of lattices and extensions of Voronoi's algorithm, in particular for lattices with a given finite group action. Finally there are numerical tables (Ch. 14) and four appendices on algebraic and arithmetic prerequisites. Readers will also appreciate the numerous exercises included, as well as the notes (at the end of each chapter) explaining the origin of the material.

Reviewer: [H.G.Quebbemann \(Oldenburg\)](#)

**MSC:**

- 11H55 Quadratic forms (reduction theory, extreme forms, etc.)
- 11-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to number theory
- 11H31 Lattice packing and covering (number-theoretic aspects)
- 52-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to convex and discrete geometry

Cited in <b>1</b> Review
Cited in <b>15</b> Documents

**Keywords:**

[Perfect lattice](#); [eutactic lattice](#); [extreme lattice](#); [Voronoi algorithm](#)