The article contains a brief review of the mutual influence of mathematics and theoretical physics in the area called supersymmetry by physicists. It is aimed at the audience of mathematicians but should be of great interest for physicists, too. The presentation is based on three examples: supersymmetric quantum mechanics, Lie superalgebras, Quillen superconnections. Presenting firstly the concise background of the formalism of the quantum mechanics, the author introduces Witten’s supersymmetric quantum mechanics and recalls its applications to differential geometry. In particular, he presents in a pedagogical way elements of the proofs of index theorems (Gauß-Bonnet, Morse) and the Morse inequalities (according to Witten). The points where supersymmetric quantum mechanics enters the proofs are clearly shown. The Lie superalgebras of finite and infinite dimension are presented as structures which are simultaneously present in a natural way in mathematics and physics. This theory is illustrated by means of examples of Lie algebra valued differential forms, the Schouten and Nijenhuis bracket and the Whitehead product. On the other side, a "physical" example includes the super-Poincaré algebra and its field representations with multiplets of the bosonic and fermionic fields. The section devoted to Quillen superconnections gives a brief description of the object and some indications to the literature on BRST symmetry. The author finishes the article by enumerating various directions of the development of theories involving $\mathbb{Z}_2$-graded structures, such as superalgebra, superanalysis, and supergeometry. The modest (85 items), yet well selected bibliography can serve as a good guidance in further reading.

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MSC:

53Z05 Applications of differential geometry to physics
17A70 Superalgebras
81T60 Supersymmetric field theories in quantum mechanics
81Q60 Supersymmetry and quantum mechanics
58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel’man theory, etc.) in infinite-dimensional spaces

Keywords:
supersymmetry; $\mathbb{Z}_2$-graded structures; supersymmetric quantum mechanics; index theorems; Lie superalgebras; supermanifolds; superconnections