

**Coron, J.-M.**

**Return method: Application to controllability.** (English) [Zbl 0872.93041](#)

Sémin. Équ. Dériv. Partielles, Éc. Polytech., Cent. Math., Palaiseau Sémin. 1992-1993, Exp. No. 14, 11 p. (1993).

This survey type publication describes the “return method” where local controllability near an equilibrium of a nonlinear control system is deduced from controllability of the linearization along a periodic trajectory constructed with a periodic feedback.

Insights in the boundary controllability of the incompressible 2-D Euler equation (on  $\Omega \subset \mathbb{R}^2$  open connected and simply connected), which was investigated by the author, are provided. The states are the velocity and the pressure which are looked in the class of smooth functions. The method consists in:

1) Extending the domain and solve a Laplace equation with appropriate boundary conditions allowing to construct a periodic trajectory (starting from the fluid at rest) for the system under consideration, so that the field at prescribed boundaries in  $\delta\Omega$  sets nonautonomously the fluid in motion in  $\Omega$ .

2) Linearizing the Euler equation about the periodic solution just obtained and showing that it is controllable. This last step is deduced from the vorticity dynamics obtained from the linearized equations; the domain is extended so that an initial vorticity can be chosen so that the boundary conditions imposed by the controllability requirement are satisfied in  $\Omega$ . Properties of harmonic functions are used.

The author does not explain why (local?) controllability of the nonlinear Euler equation follows from controllability of the linearized system along a periodic trajectory, taking into account that one considers a PDE.

There are rare typographical mistakes or imprecisions.

Reviewer: [A.Akutowicz \(Berlin\)](#)

**MSC:**

- 93C20 Control/observation systems governed by partial differential equations
- 93-02 Research exposition (monographs, survey articles) pertaining to systems and control theory
- 35Q35 PDEs in connection with fluid mechanics
- 93B05 Controllability

**Keywords:**

return method; local controllability; periodic trajectory; periodic feedback; incompressible 2-D Euler equation; Laplace equation

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