Cieliebak, K.
Symplectic boundaries: Creating and destroying closed characteristics. (English)
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In the category of symplectic manifolds with boundary, it is natural to consider the line bundle $\ker(\omega|\partial M) \to \partial M$ whose integral curves form the characteristic foliation $\mathcal{L}_\omega$. Question: Assuming only that the interiors (M^0, ω) and (M', ω') are symplectomorphic, what can one conclude about $\mathcal{L}_\omega$ and $\mathcal{L}_{\omega'}$? Eliashberg and Hofer showed that they may be not conjugate. But the author and coworkers have proved the correspondence of closed characteristics and the equality of the action spectrum. This rigidity result was proven under the assumptions that $(M, d\mu)$, $(M', d\mu')$ are compact, exact, $\mu|\partial M$, $\mu'|\partial M'$ are contact forms, and the closed characteristics are strongly nondegenerate.

In this paper, the author investigates what happens if one drops these assumptions. A geometric construction (Theorems 1 and 2) in Section 2 yields ten beautiful corollaries, which we urge Zentralblatt readers to know about. To give a glimpse, the reviewer chose Corollary I. It is related to the famous Seifert's problem on nonsingular vector fields on S^3. Paul Schweitzer proved in 1974 that there are nonsingular vector fields without periodic orbits. Seifert's question, however, may be posed for restricted types; the author considers here volume-preserving ones of confoliation type ($L_X\Omega = 0$, $i_X\Omega = d\lambda$, $\lambda(X) \geq 0$). Corollary I. Let c be any knot type in S^3. There exists a confoliation vector field with precisely two periodic orbits of knot types $\pm c$.

Reviewer: J.Koiller (Rio de Janeiro)

MSC:

53C15	General geometric structures on manifolds (almost complex, almost product structures, etc.)	Cited in **8** Documents
37J99	Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems	
53C12	Foliations (differential geometric aspects)	

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symplectic geometry; contact manifolds; foliations

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