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How good are convex hull algorithms? (English) Zbl 0877.68119

Summary: A convex polytope $P$ can be specified in two ways: as the convex hull of the vertex set $V$ of $P$, or as the intersection of the set $H$ of its facet-inducing halfspaces. The vertex enumeration problem is to compute $V$ from $H$. The facet enumeration problem is to compute $H$ from $V$. These two problems are essentially equivalent under point/hyperplane duality. They are among the central computational problems in the theory of polytopes. It is open whether they can be solved in time polynomial in $|H| + |V|$ and the dimension. In this paper we consider the main known classes of algorithms for solving these problems. We argue that they all have at least one of two weaknesses: inability to deal well with “degeneracies”, or, inability to control the sizes of intermediate results. We then introduce families of polytopes that exercise those weaknesses. Roughly speaking, fat-lattice or intricate polytopes cause algorithms with bad degeneracy handling to perform badly; dwarfed polytopes cause algorithms with bad intermediate size control to perform badly. We also present computational experience with trying to solve these problems on these hard polytopes, using various implementations of the main algorithms.

MSC:
68U05 Computer graphics; computational geometry (digital and algorithmic aspects)
52B55 Computational aspects related to convexity
65D18 Numerical aspects of computer graphics, image analysis, and computational geometry

Keywords:
convex polytope; convex hull; vertex enumeration problem; facet enumeration problem

Software:
cdd; Qhull

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References: