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On the general elephant problem for three-dimensional $\mathbb{Q}$-Fano fiber spaces over a surface.
(Russian. English summary) [Zbl 0878.14030]

Summary: We consider $\mathbb{Q}$-Fano fiber spaces $X/S$ over a surface, i.e., a three-dimensional variety $X$ with terminal $\mathbb{Q}$-factorial singularities and a projective morphism $\varphi: X \to S$ onto a normal surface $S$ such that $\varphi_* O_X = O_S$, $\rho(X/S) = 1$ and $-K_X \varphi$-ample. In this situation we discuss Reid’s conjecture on general elephants, i.e. on general members of the linear system $|-K_X + \varphi^* h|$. We prove that the surface $S$ has only cyclic quotient singularities; besides if for $X/S$ the elephants conjecture is true, then singularities of $S$ are Du Val singularities of the type $A_n$. In the last case some conditions on singularities of $X$ and $S$ are obtained.

MSC:
14J30 3-folds
14C20 Divisors, linear systems, invertible sheaves
14B05 Singularities in algebraic geometry
14J17 Singularities of surfaces or higher-dimensional varieties

Keywords:
Fano fiber spaces; three-dimensional variety; general elephants; linear system; quotient singularities; Du Val singularities