

**Gérard, Patrick; Markowich, Peter A.; Mauser, Norbert J.; Poupaud, Frédéric**  
**Homogenization limits and Wigner transforms.** (English) [Zbl 0881.35099](#)  
*Commun. Pure Appl. Math.* 50, No. 4, 323-379 (1997).

This paper deals with a theory for carrying out homogenization limits for quadratic functions  $n^\varepsilon = |u^\varepsilon(t, x)|^2$  of solutions  $u^\varepsilon$  of the following type Cauchy problems:

$$\varepsilon u_t^\varepsilon + P^\varepsilon u^\varepsilon = 0, \quad u^\varepsilon|_{t=0} = u_I^\varepsilon(x),$$

where  $\varepsilon > 0$  is a small parameter,  $\varepsilon \rightarrow 0$ ,  $u^\varepsilon \in L^2(\mathbb{R}_x^m)$ , and  $P^\varepsilon$  is an anti-selfadjoint spatial pseudodifferential operator. To do this, the authors introduce a special phase space – the space of Wigner measures and calculate them by solving some kinetic equations. The weak limits of  $n^\varepsilon$  are obtained by taking moments of the Wigner measure. Applications are given to the Schrödinger equation, to the acoustic equation in a periodic medium, and to the Dirac equation.

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**MSC:**

- [35Q40](#) PDEs in connection with quantum mechanics
- [35B27](#) Homogenization in context of PDEs; PDEs in media with periodic structure
- [35S05](#) Pseudodifferential operators as generalizations of partial differential operators

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**Keywords:**

homogenization limit; Wigner measure; Schrödinger equation; acoustic equation; Dirac equation

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