van der Poorten, Alfred J.
Notes on Fermat's last theorem. (English) Zbl 0882.11001
Canadian Mathematical Society Series of Monographs and Advanced Texts. New York, NY: Wiley. xvi, 222 p. (1996).

This marvellous book explains much of the background for the study of the modularity of elliptic curves leading to *A. Wiles*' proof of Fermat's Last Theorem [Ann. Math., II. Ser. 141, 443-551 (1995; Zbl 0823.11029)]. The book is accessible to a wide audience, while at the same time covering a considerable amount of deep and interesting mathematics. Indeed, much important technical mathematics is covered here, rigorously (though not pedantically), while at the same time with an attempt to draw the reader's attention to the flow of the argument, rather than to distract the reader with the details. I wish more mathematics books on deep topics were written so lucidly and appealingly, introducing the reader so naturally to difficult concepts.

The author does not attempt to give a complete proof of Wiles' Theorem, nor to get involved with many of the very difficult technical aspects. What he does do is give a coherent overview of the proof, digging deep enough to include some essence from the harder mathematics involved, and to introduce various fundamental questions that arise. As he writes, "My idea was to provide motive — and damn the details." Indeed, it is enough to get you started if you intend to go on to master the details.

The first few chapters of the book discuss much of the early history of number theory, in the guise of its relationship to Fermat's Last Theorem. Thus the author covers much of the material central to *P. Ribenboim*'s classic book [13 Lectures on Fermat's Last Theorem, New York: Springer Verlag (1979; Zbl 0456.10006)], but also gets to describe some of his favourite topics, such as continued fractions and p-adic numbers.

In chapter six he starts in on the modern approaches to Fermat's Last Theorem, introducing Mordell's and Faltings' Theorems, the *abc*-conjecture, and even a first shot at explaining the Birch-Swinnerton Dyer conjectures. In chapters seven and ten, he introduces elliptic functions and Weierstrass parametrizations, and some of the theory of Eisenstein series and modular forms. In the meantime in chapter nine he gives enough of the basics of reductions of elliptic curves that he can accurately explain the modularity conjecture in chapter eleven. In chapter twelve he introduces Poisson summation, and then deduces various functional equations. In chapter thirteen he gives a more detailed discussion of L-functions and their role in modern mathematics.

In chapter fifteen the author discusses heights, motivating how different notions of height are related, proceeding from Mahler measure to canonical height. This then allows him to give a more complete explanation of the Birch-Swinnerton Dyer conjectures in chapter sixteen. By describing the construction of Heegner points, the author then explains the Gross-Zagier formula, and so motivates the solution to Gauss' class number problem. In chapter seventeen, the author has a stab at explaining the relevance of Galois representations, the Deligne-Serre theorem, and thence on to his sketch of the proof of Wiles. Once you have gotten this far you are ready to move on to learning more of the details, and the author has succeeded in his goal of getting you involved in the mathematics.

Woven through the text are discussions of various other interesting recent developments in number theory, not necessarily closely related to Fermat's Last Theorem and the modularity conjecture. By developing number theory as he does, we see how these different questions arise naturally in their own context, and how these same kinds of tools lead into approaches to these various different problems. Throughout, one gets a feeling of how research mathematics is really done, the culture as well as the mathematics.

This is a wonderful book, daring to breach the stylistic barriers that usually impede understanding difficult mathematics, by being provoking, inspiring and fun. It is written to encompass a lot of material, from elementary to deep, while remaining accessible. I expect it will turn a lot of people on to number theory and arithmetic geometry, and indeed the beauty of mathematics as a whole.

Reviewer: A.Granville (Athens/Georgia)

MSC:

11-02 Research exposition (monographs, survey articles) pertaining to number theory
11D41 Higher degree equations; Fermat's equation
11-03 History of number theory
11G05 Elliptic curves over global fields
11F03 Modular and automorphic functions
11G20 Curves over finite and local fields

Keywords:

Fermat's Last Theorem; continued fractions; elliptic curves; modularity; abc-conjecture