The authors introduce the invariant $\varepsilon_M$ for definite centroaffine hypersurfaces $M^n \to \mathbb{R}^{n+1}$ in terms of the normalized scalar curvature of a plane section $\pi$ in $T_p M$. They derive the inequality

$$\varepsilon_M \geq -\frac{n^2(n-2)}{2(n-1)} h(T^\#, T^\#) + \frac{1}{2} (n+1)(n-2), \quad (*)$$

where $h$ is a positive definite metric and $T^\#$ the Tschebychev field. The class of hypersurfaces satisfying the equality in $(*)$ is a very wide one. They prove that this class consists a proper affine hyperspheres centered at the origin. Using some distribution $D$ on such hypersurfaces, the authors obtain classification theorems under the additional assumption that either $M^n$ has constant scalar curvature or the distribution $D^\perp$ is integrable. Examples related to equiaffine elliptic spheres in $\mathbb{R}^3$, realizing the equality in $(*)$ are also given.

Some results about polarization and asymptotic spectral invariants of second order Laplace type operators are related to the previously mentioned results.

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- 58J50 Spectral problems; spectral geometry; scattering theory on manifolds

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centroaffine hypersurface; curvature invariant; proper affine hyperspheres; polar hypersurfaces; asymptotic spectral geometry of Laplace type operators; Chebyshev vector field

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