Summary: A restricted-orientation convex set, also called an \( \mathcal{O} \)-convex set, is a set of points whose intersection with lines from some fixed set is empty or connected. The notion of \( \mathcal{O} \)-convexity generalizes standard convexity and orthogonal convexity. We explore some of the basic properties of \( \mathcal{O} \)-convex sets in two and higher dimensions. We also study \( \mathcal{O} \)-connected sets, which are restricted \( \mathcal{O} \)-convex sets with several special properties. We introduce and investigate restricted-orientation analogs of lines, flats, and hyperplanes, and characterize \( \mathcal{O} \)-convex and \( \mathcal{O} \)-connected sets in terms of their intersections with hyperplanes. We then explore properties of \( \mathcal{O} \)-connected curves; in particular, we show when replacing a segment of an \( \mathcal{O} \)-connected curve with a new curvilinear segment yields an \( \mathcal{O} \)-connected curve and when the catenation of several curvilinear segments forms an \( \mathcal{O} \)-connected segment. We use these results to characterize an \( \mathcal{O} \)-connected set in terms of \( \mathcal{O} \)-connected segments, joining pairs of its points, that are wholly contained in the set.

We also identify some of the major properties of standard convex sets that hold for \( \mathcal{O} \)-convexity. In particular, we establish the following results: The intersection of a collection of \( \mathcal{O} \)-convex sets in an \( \mathcal{O} \)-set; every \( \mathcal{O} \)-connected curvilinear segment is a segment of some \( \mathcal{O} \)-connected curve; for every two points of an \( \mathcal{O} \)-convex set, there is an \( \mathcal{O} \)-convex segment joining them that is wholly contained in the set.

MSC:

52A01 Axiomatic and generalized convexity
52A20 Convex sets in \( n \) dimensions (including convex hypersurfaces)

Keywords:

restricted-orientation convex set; convexity; orthogonal convexity

Full Text: DOI Link

References:

[1] Bruckner, C.K.; Bruckner, J.B., On \( L^n \)-sets, the Hausdorff metric, and connectedness, (), 765-767 · Zbl 0131.38005


[12] Montuno, D.Y.; Fournier, A., Finding the \( \text{\emph{x}} \)-\( \text{\emph{y}} \) convex hull of a set of \( \text{\emph{x}} \)-\( \text{\emph{y}} \) polygons, ()

[13] Nicholl, T.M.; Lee, D.T.; Liao, Y.Z.; Wong, C.K., Constructing the \( \text{\emph{x}} \)-\( \text{\emph{y}} \) convex hull of a set of \( \text{\emph{x}} \)-\( \text{\emph{y}} \) polygons, Bit, 23, 456-471, (1983) · Zbl 0523.68061


[18] Rawlins, G.J.E.; Wood, D., Ortho-convexity and its generalizations, (), 137-152
[21] Schuierer, S., On generalized visibility, () · Zbl 0837.68119
[23] Valentine, F.A., Local convexity and \textit{L}-sets, (), 1305-1310 · Zbl 0135.40702

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.