

**Friedrich, Thomas**

**Dirac operators in Riemannian geometry. With an outlook on Seiberg-Witten theory. (Dirac-Operatoren in der Riemannschen Geometrie. Mit einem Ausblick auf die Seiberg-Witten-Theorie.)** (German) [Zbl 0887.58060](#)

Advanced Lectures in Mathematics. Wiesbaden: Vieweg. xii, 207 S. (1997).

This textbook contains a detailed introduction to the Dirac operator on Riemannian  $\text{spin}^c$  manifolds. The reader should have some familiarity with the basic concepts of differential geometry and topology such as manifolds, covering spaces, and cohomology. After developing the algebraic theory of Clifford algebras, spinors, and the groups  $\text{Spin}(n)$  and  $\text{Spin}^c(n)$ , one finds a discussion of spin and  $\text{spin}^c$ -structures on manifolds and of their associated spinor bundles. The Dirac operator acting on spinors is introduced and various basic properties are derived, e.g., the Lichnerowicz formula. Chapter 4 is devoted to analytic aspects avoiding the general theory of elliptic operators. Essential self-adjointness of the Dirac operator is proved. In case of a compact base manifold, discreteness of the spectrum is discussed. The Fredholm property is proved and the Atiyah-Singer index theorem is stated without proof. One also finds a brief discussion of the associated  $\eta$  and  $\zeta$ -functions. Chapter 5 contains results about eigenvalue estimates, Killing spinors, and twistor spinors, mostly with proofs.

There are two appendices. The first one contains an introduction to the Seiberg-Witten theory of 4-manifolds. The second appendix is a collection of fundamental definitions and results (mostly without proofs) in the theory of principal bundles, connections, curvature forms, holonomy, etc.

Reviewer: [C.Bär \(Freiburg\)](#)

**MSC:**

- 58J60** Relations of PDEs with special manifold structures (Riemannian, Finsler, etc.)
- 58-01** Introductory exposition (textbooks, tutorial papers, etc.) pertaining to global analysis
- 53C25** Special Riemannian manifolds (Einstein, Sasakian, etc.)

Cited in **1** Review  
Cited in **13** Documents

**Keywords:**

[spinor](#); [Seiberg-Witten theory](#); [Killing spinor](#); [twistor spinor](#); [spin structure](#); [Dirac operator](#)