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Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations. (English)

Zbl 0890.49011

Systems and Control: Foundations and Applications. Boston, MA: Birkhäuser. xvii, 570 p. (1997).

Intended by the authors as a self contained, up-to-date and also as a rather comprehensive exposition of the theory of viscosity solutions and its applications to optimal control and differential games, this book has 587 pages and a bibliographical list of 538 items.

The main topics to be developed throughout the book are clearly presented in the first chapter on the model problem of the infinite horizon discounted regulator problem.

Chapter II is dedicated to the basic theory of continuous viscosity solutions starting with equivalent definitions, their main properties, and continuing with comparison and uniqueness results and special properties in the case of convex Hamiltonians. The basic theory in Chapter II is specialized in the next two chapters to optimal control problems with unrestricted and, respectively, restricted state space, the latter case leading to certain types of boundary value problems for Hamilton-Jacobi equations.

The necessity to consider discontinuous viscosity solutions is motivated in Chapter V by the minimum time problems lacking controllability and other examples; in this chapter, the main concepts of discontinuous viscosity solutions in the literature are discussed, compared and applied to several significant examples.

Chapter VI contains approximation and perturbations results that are fundamental for applications, in particular for numerical methods. Several asymptotic problems such as singular perturbations, penalization of state constraints and the limiting behaviour of the associated Hamilton-Jacobi equations are presented in Chapter VII.

As intended by the authors, the last chapter may be considered as an introduction in the theory of two-person zero sum differential games for which the framework of viscosity solutions seems to be highly efficient.

The book is completed by two appendices written by the authors' collaborators *M. Falcone* and *P. Soravia* containing a computational method and, respectively, a viscosity solution approach to H_∞ control.

The minimal mathematical background (limited, essentially, to advanced calculus, some very basic functional analysis and the fundamental facts about ordinary differential equations), the detailed and clear proofs, the elegant style of presentation and the sets of proposed exercises at the end of each section recommend this book, in the first place, as a lecture course for graduate students and as a manual for beginners in the field. However, this status is largely extended by the presence of many advanced topics and results, by the fairly comprehensive and up-to-date bibliography and, particularly, by the very pertinent historical and bibliographical comments at the end of each chapter.

In my oppinion, this book is yet another remarkable outcome of the brilliant Italian School of Mathematics.

Reviewer: S.Mirica (Bucureşti)

MSC:

49L20	Dynamic programming in optimal control and differential games
49-02	Research exposition (monographs, survey articles) pertaining to calculus of variations and optimal control
49L25	Viscosity solutions to Hamilton-Jacobi equations in optimal control and differential games
35F30	Boundary value problems for nonlinear first-order PDEs
91A23	Differential games (aspects of game theory)

Keywords:

Hamilton-Jacobi equation; viscosity solution; optimal control; differential game; value function; dynamic programming