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Heegner points, p -adic L -functions, and the Cherednik-Drinfeld uniformization. (English)

Zbl 0899.11029

Invent. Math. 131, No. 3, 453-491 (1998).

Let E/\mathbb{Q} be a modular elliptic curve of conductor N , and let K be an imaginary quadratic field. The analytic continuation and functional equation for the Hasse-Weil zeta function $L(E/K, s)$ can be determined by Rankin's method. When the sign of this functional equation is -1 , a Heegner point α_K is defined on $E(K)$. In the case where all the primes dividing N are split in K , the Heegner point comes from a modular curve parametrization of E , and the Gross-Zagier formula relates its Néron-Tate canonical height to the first derivative of $L(E/K, s)$ at $s = 1$.

B. Perrin-Riou [Invent. Math. 89, 455-510 (1987; Zbl 0645.14010)] obtained a p -adic analogue of the Gross-Zagier formula, expressing the p -adic height of α_K in terms of a derivative of the 2-variable p -adic L -function attached to E/K . At about the same time, *B. Mazur, J. Tate* and *J. Teitelbaum* [Invent. Math. 84, 1-48 (1986; Zbl 0699.14028)] formulated a p -adic Birch-Swinnerton-Dyer conjecture for the p -adic L -function of E associated to the cyclotomic \mathbb{Z}_p -extension of \mathbb{Q} . In an earlier paper, *M. Bertolini* and *H. Darmon* [Invent. Math. 126, 413-456 (1996; Zbl 0882.11034)] proposed analogues of the Mazur-Tate-Teitelbaum conjectures for the p -adic L -function of E associated to the anticyclotomic \mathbb{Z}_p -extension of K . In a special case, they predicted a p -adic analytic construction of the Heegner point α_K from the first derivative of the anticyclotomic p -adic L -function. This paper provides a proof of this conjecture.

Reviewer: [Min Ho Lee \(Cedar Falls\)](#)

MSC:

- 11G40** L -functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture
- 14G10** Zeta functions and related questions in algebraic geometry (e.g., Birch-Swinnerton-Dyer conjecture)
- 11G05** Elliptic curves over global fields
- 11F67** Special values of automorphic L -series, periods of automorphic forms, cohomology, modular symbols

Cited in **6** Reviews
Cited in **21** Documents

Keywords:

modular elliptic curves; p -adic Birch-Swinnerton-Dyer conjecture; Heegner points; p -adic L -functions; Cherednik-Drinfeld uniformization; Mazur-Tate-Teitelbaum conjectures

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