Jungck, G.; **Rhoades, B. E.**

Fixed points for set valued functions without continuity. (English) Zbl 0904.54034

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Let (X,d) be a metric space and $\mathcal{B}(X)$ the set of all nonempty bounded subsets of X. If $A,B\in\mathcal{B}(X)$ then $\delta(A,B)=\{\sup d(a,b)\colon a\in A \text{ and } b\in B\}$ and $d(x,B)=\inf\{d(a,b): b\subset B\}$. The authors extend the concepts of compatible (weak) maps and generalized Meir-Keeler contractions.

Definition 1. Let (X,d) be a metric space and let $A,B: X\to\mathcal{B}(X)$. Then A and B are $(\varepsilon,\gamma)(p)$ contractions relative to maps $S,T: X\to X$ if $\bigcup A(X)\subset T(X), \bigcup B(X)\subset S(X)$, and there exist functions $p: X\times X\to[0,\infty)$, $\gamma:(0,\infty)\to(0,\infty)$ such that $\gamma(\varepsilon)>\varepsilon$ for all ε and for $x,y\in X$:

$$0<\varepsilon<p(x,y)<\gamma(\varepsilon)\Rightarrow\delta(Ax,By)<\varepsilon.$$

Definition 2. Let (X,d) be a metric space and let $S: X\to X$ and $A: X\to\mathcal{B}(X)$. The pair $\{A,S\}$ is a weakly compatible pair if $Ax=\{Sx\}$ implies $SAx=ASx$. Every compatible pair $\{A,S\}$ [*G. Jungck* and *B. E. Rhoades*, Int. J. Math. Math. Sci. 16, No. 3, 417-428 (1993; Zbl 0783.54038)] is weakly compatible. Examples of weakly compatible pairs which are not compatible are given in the paper. In this paper the authors prove some fixed point theorems for set valued functions without appeal to continuity. These theorems extend results of *T.-H. Chang* [Math. Jap. 38, No. 4, 675-690 (1993; Zbl 0805.47049)], generalize results by *J. Jachymski* [ibid. 42, No. 1, 131-136 (1995; Zbl 0845.47044)] and by *S. M. Kang* and *B. E. Rhoades* [ibid. 37, No. 6, 1053-1059 (1992; Zbl 0767.54037)] and produce as byproducts generalizations of theorems for point valued functions.

Theorem 4.1: Let S and T be self maps of a metric space (X,d) and let $A,B: X\to\mathcal{B}(X)$. Suppose $\bigcup A(X)\subset T(X)$, $\bigcup B(X)\subset S(X)$, and one of $S(X)$, $T(X)$ is complete. Let $p: X\times X\to[0,\infty)$ and φ: $[0,\infty)\to(0,\infty)$ be maps, and suppose that $\varphi(t)<t$ for $t>0$. If $\delta(Ax,By)\le\varphi(p(x,y))$ for $x,y\in X$, then there exists a unique point $z\in X$ such that $\{z\}=\{Sz\}=\{Tz\}=Az=Bz$ provided that both $\{A,S\}$ and $\{B,T\}$ are weakly compatible pairs, and one of (a), (b) below is true:

(a) $p=\max\{d(Sx,Ty),\frac{1}{2}(d(Ax,Ty)+d(Sx,By))\}$, $\delta(Ax,By)=0$, whenever $m=0$, and φ is u.s.c. from the right; and

(b) $p=\max\{d(Sx,Ty),\ \delta(Ax,Sx),\ \delta(By,Ty),\ \frac{1}{2}(d(Ax,Ty)+d(Sx,By))\}$ and φ is u.s.c.

Reviewer: V.Popa (Bacau)

MSC:

54H25 Fixed-point and coincidence theorems (topological aspects)
47H10 Fixed-point theorems
54C60 Set-valued maps in general topology