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Asymptotic behavior of global solutions to the Navier-Stokes equations in $\mathbb{R}^3$. (English)

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The author considers the Cauchy problem to the Navier-Stokes equations

$$\frac{\partial u}{\partial t} - \Delta u + (u \cdot \nabla)u + \nabla p = 0, \qquad \text{div } u = 0, \quad x \in \mathbb{R}^3,\ t > 0, \tag{1}$$

$$u(x,0) = u_0(x), \qquad x \in \mathbb{R}^3.$$

It is proved that if the norm of u_0 in Besov space is sufficiently small then the problem has a global unique solution. Another result is related to the asymptotic behavior of the above solution. Let $u(x,t)$ be a solution of the problem (1) and $v(x,t) = \lim_{\lambda\to\infty} \lambda u(\lambda x, \lambda^2 t)$ (it corresponds to replacing x by $x/\sqrt{t}$ and tending $t \to \infty$). If $v(x,t)$ is the solution of the Navier-Stokes system with initial data $v_0(x) = \lim_{\lambda\to\infty} \lambda u(\lambda x, 0)$, then $v(x,t)$ is a self-similar solution. The following result is proved: If $\sqrt{t}u(\sqrt{t}x, t) \to V(x)$ in L^p as $t \to \infty$ for $p \in (3, +\infty)$ then $v(x,t) = (1/\sqrt{t})V(x/\sqrt{t})$ is a self-similar solution of (1).

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MSC:

35Q30 Navier-Stokes equations
35B40 Asymptotic behavior of solutions to PDEs

Keywords:

Cauchy problem; self-similar solution

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