Joyce, Dominic

Compact manifolds with exceptional holonomy. (English) Zbl 0911.53021

Doc. Math., Extra Vol. ICM Berlin 1998, vol. II, 361-370 (1998).

The author describes (without giving details) his famous construction of compact Riemannian 7-manifolds M with holonomy group G_2, which is an imitation of the Kummer construction of a K3 surface as a desingularization of the orbifold $T^4/\mathbb{Z}_2$. A G_2-structure on a 7-manifold M can be defined as a pair (g, ϕ) where g is a Riemannian metric and ϕ is a 3-form of special type. The tensor $T = \nabla\phi$ is called the torsion of the G_2-structure (g, ϕ). If the torsion $T = 0$ and the fundamental group of M is finite, then the metric g has the holonomy group $\mathrm{Hol}(g) \subset G_2$. The construction of compact Riemannian manifolds with holonomy G_2 consists of four steps:

Let T^7 be the 7-torus with a flat G_2-structure (g_0, ϕ_0). The author chooses some finite group Γ of automorphisms of (g_0, ϕ_0) and considers the orbifold T^7/Γ.

Using complex geometry and results by P. B. Kronheimer, the author resolves the singularities of the orbifold and gets a smooth compact 7-manifold M with a map $\pi : M \to T^7/\Gamma$, the resolving map.

He constructs a 1-parameter family (g_t, ϕ_t), $t \in (0, \varepsilon)$, of G_2-structures on M, such that the torsion T_t becomes small when $t \to 0$.

Using analysis, the author proves that a G_2-structure with sufficiently small torsion can be deformed to a G_2-structure without torsion. This implies existence of a metric with holonomy G_2 on M. Some information about 68 compact manifolds M with holonomy G_2 which can be obtained by this method is given. In particular, a graph of their Betti numbers b_2, b_3 is presented.

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MSC:

53C20 Global Riemannian geometry, including pinching
58J60 Relations of PDEs with special manifold structures (Riemannian, Finsler, etc.)
53C15 General geometric structures on manifolds (almost complex, almost product structures, etc.)
53C25 Special Riemannian manifolds (Einstein, Sasakian, etc.)

Keywords:

exceptional holonomy; G_2-structure on a manifold; Kummer construction; resolution of singularities; desingularisation

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