Consider a metaplectic central extension $1 \to A \to \tilde{G} \to G$, where $G = GL(n)$ over a field. The author looks at liftings of the standard outer involution $\iota$ of $GL(n)$. Recall that $\iota(g) = w_0 t g t^{-1} w_0^{-1}$ sends diagonal matrices to diagonal matrices and upper triangular matrices to upper triangular ones. To the metaplectic extension a 2-cocycle class $[\tau] \in H^2(GL(n), A)$ is associated, which is known to be invariant under $\iota$. For explicit computations one would like to use a cocycle $\tau$ representing this class $[\tau]$. It was mistakenly claimed by D. A. Kazhdan and S. J. Patterson [in: “Metaplectic forms”, Publ. Math., Inst. Hautes Étud. Sci. 59, 35-142 (1984; Zbl 0559.10026)] that one may choose $\tau$ to be invariant under $\iota$. On the perfect subgroup $SL(n)$ one can try a Steinberg–Matsumoto cocycle, and hope it to be invariant. In any case, using the remaining $GL(1)$, the author finds an obstruction to invariance of $\tau$ under $\iota$. Next he treats the case of topological central extensions. The results are then used to lift $\iota$ to a continuous automorphism of $\tilde{G}$ in the case of most frequent interest.

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