van der Hoeven, Joris

Fast evaluation of holonomic functions. (English) Zbl 0912.68081

Theor. Comput. Sci. 210, No. 1, 199-215 (1999).

Summary: A holonomic function is an analytic function, which satisfies a linear differential equation with polynomial coefficients. In particular, the elementary functions exp, log, sin, etc. and many special functions like erf, Si, Bessel functions, etc. are holonomic functions. Given a holonomic function f (determined by the linear differential equation it satisfies and initial conditions in a non singular point z), we show how to perform arbitrary precision evaluations of f at a non singular point z' on the Riemann surface of f, while estimating the error. Moreover, if the coefficients of the polynomials in the equation for f are algebraic numbers, then our algorithm is asymptotically very fast: if $M(n)$ is the time needed to multiply two n digit numbers, then we need a time $O(M(n\log^2 n\log\log n))$ to compute n digits of $f(z')$.

MSC:

68W30 Symbolic computation and algebraic computation

68W10 Parallel algorithms in computer science

Keywords:

holonomic function

Full Text: DOI

References:

[1] Borwein, J.M.; Borwein, P.B., Pi and the AGM, (1987), Wiley New York · Zbl 0699.10044

[2] Brent, R.P., The complexity of multiprecision arithmetic, ()

[3] Brent, R.P., Fast multiple-precision evaluation of elementary functions, J. ACM, 23, 2, 242-251, (1976) · Zbl 0324.65018

[4] Brent, R.P.; MacMillan, E., Some new algorithms for high-precision computation of Euler's constant, J. ACM, 23, 242-251, (1980)

[5] Haible, B.; Papanikolaou, T., Fast multiple-precision evaluation of elementary functions, (), Available on the web at

[6] van der Hoeven, J., Fast evaluation of holonomic functions, (), Available on the web at · Zbl 0912.68081

[7] van der Hoeven, J., Automatic asymptotics, () · Zbl 0874.65047

[8] Knuth, D.E., The art of computer programming, () · Zbl 0191.17903

[9] Lipshitz, L., D-finite power series, J. algebra, 122, 353-373, (1989) · Zbl 0695.12018

[10] Naegele, F., Autour de quelques équations fonctionnelles analytiques, ()

[11] Richardson, D., How to recognize zero, J. symbolic computat., 24, 627-645, (1997) · Zbl 0917.11062

[12] Salamin, E., Computation of π using the arithmetic-geometric Mean, Math. comput., 30, 565-570, (1976) · Zbl 0345.10003

[13] Schönhage, A.; Strassen, V., Schnelle multiplikation grosser zahlen, Computing, 7, 281-292, (1971) · Zbl 0223.68007

[14] Stanley, R.P., Differentially finite power series, Eur. J. combin., 1, 175-188, (1980), MR # 81m:05012. · Zbl 0445.05012

[15] Thomann, J., Procédés formels et numériques de sommation de séries d'équations différentielles, Expositiones math., 13, 223-246, (1995) · Zbl 0831.34003

[16] Zeilberger, D., A holonomic systems approach to special functions identities, J. comput. appl. math., 32, 321-368, (1990) · Zbl 0738.33001