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Bouche, Thierry

Asymptotic results for hermitian line bundles over complex manifolds: The heat kernel approach. (English) Zbl 0914.32010

Andreatta, Marco (ed.) et al., Higher dimensional complex varieties. Proceedings of the international conference, Trento, Italy, June 15–24, 1994. Berlin: Walter de Gruyter. 67-81 (1996).

This paper is intended to be an introduction to a heat kernel approach to some problems in complex geometry. Aim is to indicate an entirely new method of constructing holomorphic (or more generally harmonic) sections of vector bundles over complex manifolds by using heat kernel estimates. Let X is a compact complex analytic manifold of dimension n, endowed with a hermitian metric ω and associated volume element $dV = \frac{\omega^n}{n!}$, E (resp. L) is a hermitian holomorphic vector bundle of rank r (resp. 1). For a global C^∞ $(0,q)-$ form σ of $E(k) = E \otimes L^{\otimes k}$, $|\sigma(x)|^2$ denotes the pointwise length induced by the given metrics.

Theorem 1.2. For any $q = 0, \dots, n$, $\dim H^q(X.E(k)) \leq (-1)^q r \frac{k^n}{n!} \int_{X(q)} (\frac{i}{2\pi} c(L))^n + o(k^n)$.

Theorem 1.3 (generalized Kodaira vanishing). If the curvature of L has at least $n - q + 1$ positive eigenvalues everywhere on X, then $H^i(X, E(k)) = 0$ for $i \geq q$ as soon as k is sufficiently large. Let the family $h_1, \dots, h_m$ be an L^2 orthonormal basis of the space $\mathcal{H}^q(X, E(k))$ of harmonic $(0,q)-$ forms, then the distortion function is defined by: $b_k^q(x) = \sum_{j=1}^m |h_j(x)|^2$.

Theorem 2.1. If the curvature of L has constant signature $q(i.e. X(q) = X)$ then the following asymptotic estimation holds: $b_k^q \sim r(-1)^q \det_\omega \frac{i}{2\pi} c(L)_x (k^n + o(k^n))$ uniformly over X.

Theorem 3.1. Fix $x_0 \in X$. Then, for any $k \geq 0$ there exists a global section σ_k of $E(k)$ of unit L^2 norm such that for any geodesic ball of radius r_k such that $r_k\sqrt{k} \to +\infty$, the following bound holds $\int_{B(x_0,r_k)} |\sigma_k|^2 \, dV \geq 1 - o(1)$. Where $o(1)$ means some quantity tending to zero when $k \to +\infty$. Moreover, $|\sigma_k(x)|^2 = o(k^n)$ uniformly on compact subsets of $X \backslash \{x_0\}$.

Corollary 3.1 (Kodaira embedding theorem). If L is positive, the map $\Phi_{kL} : X \to \mathbf{P}(H^0(X, L^k)^*)$ defined by the linear system $|kL|$ is a projective embedding for k sufficiently large.

For the entire collection see [Zbl 0859.00021].

Reviewer: V.V.Chueshev (Kemerovo)

MSC:

32L05	Holomorphic bundles and generalizations
32L10	Sheaves and cohomology of sections of holomorphic vector bundles, general results
58J35	Heat and other parabolic equation methods for PDEs on manifolds

Cited in **3** Reviews
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Keywords:

heat kernel; positive line bundle; Kodaira theorems; distortion function

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