Consider the scalar initial value problem (IPV) in the closed finite interval \([a, b]\) as follows:

\[
P(D)y(x) = f(x, y), \quad D = \frac{d}{dx}, \quad y(a) = y_0, \ldots, y^{(m-1)}(a) = y^{(m-1)}_0,
\]

where \(f\) is a continuous function with continuous derivatives till a certain order \(m + k\) and satisfying all conditions for the existence and uniqueness of solutions. \(P(D)\) is the operator \((D - \lambda_1) \cdots (D - \lambda_m)\), \(\lambda_1, \ldots, \lambda_m\) are real or complex conjugate constants. The paper presents a general procedure allowing to construct multistep methods with coefficients depending on the step for the exact integration of both homogeneous linear problem and nonhomogenous IVP (1) without truncation error. \(f\) is supposed to be a certain Fourier polynomial in one or more frequencies resulting in oscillator solution. Such an algorithm is denoted as adapted to (1). It combines the integration of both the linear part and a particular solution of the complete problem.

The study of methods adapted to linear equations is presented in a unified way especially with respect to consistency and convergence. The results are valid for all presented examples and different existing methods. The adaptation of a multistep code to the exact solution of a forced oscillator in one frequency includes a new procedure for the computation of coefficients. The exact integration of oscillators in two or more frequencies which are confluent or non-confluent include general formulae for the calculation of the coefficients and the local truncation error. Numerical examples illustrate efficiency of the derived results. Some of the ideas are related to the adaptation of the Chebyshev methods without multistep character by J. Panovsky and D. L. Richardson [J. Comput. Appl. Math. 23, 35-51 (1988; Zbl 0649.65048)].

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MSC:

65L06 Multistep, Runge-Kutta and extrapolation methods for ordinary differential equations
65L05 Numerical methods for initial value problems involving ordinary differential equations
34A34 Nonlinear ordinary differential equations and systems
34C10 Oscillation theory, zeros, disconjugacy and comparison theory for ordinary differential equations

Keywords:

consistency; multistep methods; convergence; oscillatory problems; forced oscillator; numerical examples; Chebyshev methods

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