Innamorati, Stefano

The non-existence of certain large minimal blocking sets. (English) Zbl 0917.51012

Mitt. Math. Semin. Gießen 235, 1-23 (1998).

The paper under review is concerned with the size of minimal blocking sets. In a projective plane Π_q of order q, a blocking k-set is a set of k elements which meets every line but contains no line completely. A blocking set $\mathcal{B}$ is called minimal iff for each $P \in \mathcal{B}$, $\mathcal{B} \setminus \{P\}$ is not a blocking set. A basic problem in Finite Geometry is to find the values of k for which a minimal blocking k-set (in a given projective plane) exists. For a nice survey on blocking sets see *A. Blokhuis* [Bolyai Soc. Math. Stud. 2, 133-155 (1996; Zbl 0849.51005)].

Let $\mathcal{B}$ be a minimal blocking k-set in Π_q. It is known that (1) $q+\sqrt{q}+1 \leq k \leq q\sqrt{q}+1$; (2) $k = q+\sqrt{q}+1$ iff q is a square and $\mathcal{B}$ is a Baer plane; (3) $k = q\sqrt{q}+1$ iff q is a square and $\mathcal{B}$ is a unital [see *A. A. Bruen*, Bull. Am. Math. Soc. 76, 342-344 (1970; Zbl 0207.02601), SIAM J. Appl. Math. 21, 380-392 (1971; Zbl 0252.05014), *A. A. Bruen* and *J. A. Thas*, Geom. Dedicata 6, 193-203 (1977; Zbl 0367.05009)]. If, in addition, the plane Π_q is Desarguesian and $q \geq 16$ is a square, then there do not exist minimal blocking k-sets with $q+\sqrt{q}+2 \leq k \leq q+2\sqrt{q}$ [see *S. Ball* and *A. Blokhuis*, Finite Fields Appl. 2, 125-137 (1996; Zbl 0896.51008)]. If $q = 9$ the foregoing holds with $k = 14$, by a result of *A. A. Bruen* and *R. Silverman* [Eur. J. Comb. 8, 351-356 (1987; Zbl 0638.51013)]. However it is not known whether or not there exists a minimal blocking 15-set in Π_9.

Next step is to look at large minimal blocking k-sets with k close to the upper bound $q\sqrt{q}+1$. *A. Blokhuis* and *K. Metsch* [Math. Soc. Lect. Note. Ser. 191, 37-52 (1993; Zbl 0797.51012)] noticed that there does not exist a blocking $q\sqrt{q}$-set in Desarguesian projective planes of square order $q \geq 25$.

In the paper under review the author extends this result to an arbitrary projective plane of order 9. The main ingredients of the proof are standard diophantine equations; inner and outer point equations associated to blocking sets; a previous result of the author and A. Maturo concerning blocking sets in Π_7 ; and a case by case elimination of 19 possible cases. Finally, the case $q = 16$ of Blokhuis and Metsch's result remains open and it seems difficult to extend the techniques of this paper to this case.

Reviewer: Fernando Torres (Campinas)

MSC:

51E20 Combinatorial structures in finite projective spaces

51E21 Blocking sets, ovals, k-arcs

Cited in **1** Document

Keywords:

blocking sets; Desarguesian projective planes

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