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Dynamics of Ginzburg-Landau vortices. (English) Zbl 0923.35167

Arch. Ration. Mech. Anal. 142, No. 2, 99-125 (1998).

The authors study the asymptotics of the sequence of complex valued solutions $u^\varepsilon(x,t)$ in the limit $\varepsilon \to 0$ of the system

$$\partial u^\varepsilon/\partial t - \Delta u^\varepsilon = \varepsilon^{-2} u^\varepsilon (1 - |u^\varepsilon|^2) \quad \text{in } \Omega \times (0,\infty),$$

$$u^\varepsilon(x,t) = g(x) \quad \text{on } \partial\Omega \times (0,\infty).$$

Here $\Omega \subset \mathbb{R}^2$ is an open bounded subset and g is a given function with $|g| = 1$. The system provides a gradient flow of the functional

$$I^\varepsilon(w) = \int_\Omega (\tfrac{1}{2}|\nabla w|^2 + \varepsilon^{-2}\tfrac{1}{4}(1-|w|^2))dx$$

which serves for the main technical tool. The most interesting geometrical results concern the behaviour of vortices (the zeroes of solutions). Assuming that initially there are N isolated vortices with degree ± 1, then, in the limit, these vortices persist and satisfy a system of ordinary differential equations of the kind

$$\frac{d}{dt} y^i(t) = -2d_i \left((\nabla\varphi(y^i(t), \vec{y}(t)))^\perp + \sum_{m\neq i} d_m \frac{y^m(t) - y^i(t)}{|y^m(t) - y^i(t)|^2} \right),$$

where $d_i \in \{\pm 1\}$, φ is a solution of a Dirichlet problem (which cannot be stated here), $\perp$ denotes an orthogonal vector.

All the proofs are given with details and are completed by numerous comments.

Reviewer: Jan Chrastina (Brno)

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)

35K57 Reaction-diffusion equations

Keywords:

reaction diffusion system; nodal set; vortex; Ginzburg-Landau model

Full Text: DOI