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Bipartite distance-regular graphs. I. (English) Zbl 0927.05083

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Let $Y=(X,\{R_i\}_{0\le i\le D})$ be a bipartite P- and Q-polynomial association scheme. Then $A_0,\dots,A_D$ form a basis for the Bose-Mesner algebra M of Y (A_i is the ith associate matrix of Y). We now recall the dual Bose-Mesner algebra M^* of Y. Fix any $x\in X$. For integer i ($0\le i\le D$) let $E_i^*=E_i^*(x)$ denote the diagonal matrix with the rows and columns indexed by X and $(E_i^*)yy=1$, if $xy\in R_i$, and $(E_i^*)yy=0$, if $xy\notin R_i$. We refer to E_i^* as the ith dual idempotent of Y with respect to x. It follows that the matrices $E_0^*,\dots,E_D^*$ form a basis for a algebra $M^*=M^*(x)$ known as the dual Bose-Mesner algebra of Y with respect to x. The subalgebra $T=T(x)$ of $\mathrm{Mat}_X(\mathbb{C})$ generated by M and M^* is called the Terwilliger algebra of Y with respect to x. Let V denote the vector space $\mathbb{C}^X$ (column vectors). Then $\mathrm{Mat}_X(\mathbb{C})$ acts on $\mathbb{C}^X$ by left multiplication. By a T-module, we mean a subspace W of V such that $TW\subseteq W$. An irreducible T-module W is said to be thin whenever $\dim E_i^*(W)\le 1$ for $0\le i\le D$. The endpoint of W is $\min\{i\mid E_i^*(W)\ne 0\}$. Let $\Gamma=(X,E)$ denote a bipartite distance-regular graph with diameter $D\ge 4$ and fix a vertex x of Γ. In this paper the structure of the (unique) irreducible T-module of endpoint 0 is determined. Up to isomorphism there is a unique irreducible T-module of endpoint 1. It is thin. Determined are the structure of each thin irreducible T-module W of endpoint 1 or 2 in terms of the intersection numbers of Γ and an additional real parameter $\psi=\psi(W)$ in case the endpoint is 2, which the author refers to as the type of W. It is assumed that each irreducible T-module of endpoint 2 is thin and the following two-fold result is obtained. First, the intersection numbers of Γ are determined by the diameter D of Γ and the set $\{(\psi,\mathrm{mult}(\psi))\mid \psi\Phi_2\}$, where Φ_2 denotes the set of distinct types of irreducible T-modules with endpoint 2, and where $\mathrm{mult}(\psi)$ denotes the the multiplicity with which the module of type ψ appears in the standard module V. Secondly, the set $\{(\psi,\mathrm{mult}(\psi))\mid \psi\Phi_2\}$ is determined by the intersection numbers k, b_2, b_3 of Γ and the spectrum of the graph $\Gamma_2^2=\Gamma_2(\Gamma_2(x))$.

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