Singh, Simon

Fermat's Last Theorem. The adventurous history of a mathematical riddle. (Fermats letzter Satz. Die abenteuerliche Geschichte eines mathematischen Rätsels. Aus dem Englischen von Klaus Fritz.) (German) Zbl 0930.00002

München: Carl Hanser Verlag. 364 S. (1998).

This book mainly is directed to non-mathematicians though it is also a pleasant reading for mathematicians. As the title indicates the story deals with the development of Fermat's Last Theorem and the many and finally successful efforts to prove it.

However, it would be an understatement just to characterize the content by what is given by the English subtitle. The author succeeded in presenting to a wider audience a comprehensive view of the development of mathematical reasoning from ancient times to modern use of computational devices, of the varying social background for doing mathematical research and of the different kinds of behaviour how mathematicians communicate and share their achievements. It has always been a problem for mathematicians to explain what they are doing and why these things are of interest. The main attraction of this book is how the author covered this gap, though without any reservation the history of Fermat's Last Theorem and its proof is an adventure as such.

The non-mathematician will mainly notice that the book is an easy reading about mathematics which goes beyond the usually presented simple things. He admittedly will have to invest some efforts to understand several of the mathematical details, but he always has a good chance to succeed. The mathematician will be confronted with an interesting outsider's view of the mathematical community which reveals a lot of characteristic details about the relations within this group.

The author does not have the ambition to present a work that satisfies the requirements of a scientific text in the history of mathematics. Moreover, some mathematical details will not find the full approval of all experts on the subject. But the mainstream of the historical background of Fermat's Last Theorem and the developments related to the search for its proof are well-represented and based on solid and comparatively profound investigations and interviews by the author.

The historical stations the reader will be invited to visit can be summarized as follows: The Pythagorean school of mathematics and a first description of mathematical reasoning; Euclid's work and the Arithmetica of Diophantos; mathematics at Fermat's time, his special way of communication with other mathematicians and the influence of the Arithmetica on his work; first successful steps to recover the proof of his Last Theorem by Euler, and later on by Sophie Germain; the competing "proofs" by Lamé and Cauchy and how Kummer revealed the gaps in them; the Wolfskehl prize and the inflation of wrong proofs by amateurs; the period of skepticism to find a proof initiated by the work of Russell and Gödel; the relations of number theory with coding theory; Andrew Wiles' first research on elliptic curves under the guidance of John Coates; the Taniyama-Shimura conjecture; the Frey-Ribet bridge, coupling this conjecture with Fermat's Last Theorem; the long period of Wiles' isolated efforts to prove the Taniyama-Shimura conjecture, accompanied by a description of the work of Galois and how his methods (among others) contributed to a first success to find a proof; the failure of the competing proof by Miyaoka; the essential gap in the proof detected by Katz and the cooperation between Wiles and Taylor to repair this.

As is made clear from the beginning of the book, Wiles succeeded in repairing the gap in 1994 and the full proof of Fermat's Last Theorem has been published after a period of thorough peer-reviewing in the Annals of Mathematics in 1995 [Ann. Math. (2) 141, 443-551 (1995; Zbl 0823.11029), 141, 553-572 (1995; Zbl 0823.11030)].

In a final paragraph the development of this proof is compared with the ambitions to solve two other long-standing conjectures: the four-colour-problem and Kepler's conjecture. The first one had been solved by using a computer providing a proof which could not be checked by a human being. The second one, which deals with the densest packing of balls in space, serves as an example where a controversy developed, whether a proof with gaps should be considered as a solution or not, or what kind of gaps could be accepted. If Hsiang's "proof" had not been promoted by a well-established mathematician like him, nobody would have considered this as a proof just from the beginning. The author gives quite a fair description of this situation, based on comments of several experts. In the given context this just should show that to have or not to have a proof is not always an unanimous decision. By the way, there are promising indications that meanwhile Hales has found a proof of the Kepler conjecture that will be accepted by a wider audience.

For reviews of the English and American editions see Zbl 0930.00001 and Zbl 0913.01003.

Reviewer: Bernd Wegner (Berlin)

MSC:

00A06 Mathematics for nonmathematicians (engineering, social sciences, etc.)
11-03 History of number theory
01A70 Biographies, obituaries, personalia, bibliographies
01A05 General histories, source books

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