Arhangel'skii, A. V.; **Buzyakova, R. Z.**

On linearly Lindelöf and strongly discretely Lindelöf spaces. (English) Zbl 0930.54003

A topological space X is said to be linearly Lindelöf if every open cover of X which is a chain (under set-theoretic inclusion) admits a countable subcover. It has been known for many years [*A. S. Mishchenko*, Sov. Math., Dokl. 3, 1199-1202 (1962); translation from Dokl. Akad. Nauk SSSR 145, 1224-1227 (1962; Zbl 0121.17501)] that a linearly Lindelöf Tychonoff space need not be Lindelöf. (The present authors contribute another example, discovered independently by G. Gruenhage and R. Buzyakova, which "seems to be simpler than Mishchenko's example". This has the pleasing feature that it is a dense subgroup of $\{0,1\}^{\aleph_\omega}$, hence is homogeneous and of countable cellularity.)

In 1969 the first-listed co-author established a famous theorem of which the following may be viewed a the special case $\kappa = \omega$: Every first countable Lindelöf Tychonoff space X satisfies $|X| \le 2^\omega$. Perhaps the most striking result of the present paper is that the conclusion holds also if the hypothesis "Lindelöf" is weakened to "weakly Lindelöf". A principal lemma is the following unexpected result: If Y is a closed subset of a linearly Lindelöf Tychonoff space X such that (a) $|Y| \le 2^\omega$, (b) $\chi(p, X) \le \omega$ for each $p \in Y$, and (c) every open cover of X admits a subcover of cardinality $\le (2^\omega)^+$, then Y is the intersection of 2^ω-many open subsets of X.

The paper offers (for several properties P) theorems of the form "linearly Lindelöf+$P \Rightarrow$Lindelöf". There are several unsolved problems, including: Does "linearly Lindelöf+$P \Rightarrow$Lindelöf" when P is one of these properties: Hewitt-complete (realcompact); locally compact; locally metrizable?

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MSC:

54A25	Cardinality properties (cardinal functions and inequalities, discrete subsets)
54D20	Noncompact covering properties (paracompact, Lindelöf, etc.)
54A35	Consistency and independence results in general topology

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Keywords:

Lindelöf space; linearly Lindelöf space; free sequence

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