Burns, Daniel; **Gong, Xianghong**
Singular Levi-flat real analytic hypersurfaces. (English) Zbl 0931.32009
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Let M be a real analytic hypersurface in $\mathbb{C}^n$ defined by a real analytic function r. First, we assume that M is smooth. Then, M is called Levi-flat, if it is foliated by smooth holomorphic hypersurfaces in $\mathbb{C}^n$. More precisely, for a point $p \in M$ with $dr(p) \neq 0$, M is Levi-flat near p if and only if the rank of the Levi-matrix $\left(\begin{smallmatrix} 0 & r_{\overline{z}} \\ r_z & r_{z\overline{z}} \end{smallmatrix}\right)$ is equal to 2.

In case where M has a singularity, we define the concept of Levi-flat in the following manner: Let M^* denote the smooth locus of M, i.e., the set of points $p \in M$ such that near p M is real analytically isomorphic to $\mathbb{R}^{2n-1}$. If M^* is Levi-flat, then we say that M is Levi-flat. Singular Levi-flat real analytic sets occur as invariant sets of integrable holomorphic Hamiltonian systems.

This article establishes basic theory of singular Levi-flat real analytic hypersurfaces at double points. There are two main results. Let $z_1, z_2, \dots, z_n$ denote the complex coordinates on $\mathbb{C}^n$. Let M be a Levi-flat real analytic hypersurface defined by a function in the form $q(z) + H(z)$, where $q(z)$ is a real valued quadratic form in $\mathrm{Re}(z_1), \mathrm{Re}(z_2), \dots \mathrm{Re}(z_n)$, $\mathrm{Im}(z_1), \mathrm{Im}(z_2), \dots, \mathrm{Im}(z_n)$, and $H(z)$ is a real analytic function with $H(z) = O(|z|^3)$.

Theorem 1.1: Assume that $q(z) = \mathrm{Re}(z_1^2 + z_2^2 + \dots + z_n^2)$. Then, near 0 M is biholomorphically equivalent to the cone defined by $\mathrm{Re}(z_1^2 + z_2^2 + \dots + z_n^2)$.

Theorem 1.2: Assume that $q(z)$ is positive definite on a complex line and its Levi-matrix has rank at least 2 at 0. Then, near 0 M is biholomorphically equivalent to the cone defined by $z_1\overline{z}_1 - z_2\overline{z}_2$.

Besides the above two results, in the article we can find a lot of basic results on the subject.

Reviewer: Tohsuke Urabe (Ibaraki)

MSC:

32V40	Real submanifolds in complex manifolds
32S25	Complex surface and hypersurface singularities

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