Kisin, Mark

Local constancy in p-adic families of Galois representations. (English) Zbl 0932.32028
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Let K be a field complete with respect to a non-archimedean norm, of residue characteristic p. Let X be a quasi-compact rigid space and Z a closed subspace.

A substantial part of the paper deals with the question when there exists a relative tubular neighbourhood U of Z in X. This essentially means that U is an admissible open containing Z and any Galois prime to p covering $\mathcal{E}$ of X whose restriction to Z is split, is also split over U, possibly after a finite extension of scalars. If X is smooth and Z has normal crossings, then such a relative tubular neighbourhood exists. The author therefore conjectures a rigid analogue of de Jong's resolution of singularities [*A. J. de Jong*, Publ. Math., Inst. Hautes Etud. Sci. 83, 51-93 (1996; Zbl 0916.14005)], and then shows that under this hypothesis, any pair (X, Z) admits a relative tubular neighbourhood. In particular, if the characteristic of K is zero or if X is the analytization of a scheme of finite type over K, then no assumption is needed. The main application of this work is the following. Let X be a scheme of finite type over K and let $\mathcal{L}$ be a local system which is either induced by an etale covering of X or is a lisse sheaf of free $\mathbf{Z}_l$-modules (with $l \neq p$). For each rational point $x \in X$, let

$$\rho_x \operatorname{Gal}(\overline{K}/K) \to \operatorname{Aut}(\mathcal{L}_x)$$

be the induced representation of the absolute Galois group. Then there exists a p-adic neighbourhood U of x, such that for each rational point $y \in U$, we have $\rho_x \cong \rho_y$. In particular, if F is discretely valued with finite residue field, then only finitely many non-isomorphic Galois representations can arise.

Reviewer: Hans Schoutens (Middletown)

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