Frieze, Alan; **Kannan, Ravi**

Quick approximation to matrices and applications. (English) Zbl 0933.68061

Combinatorica 19, No. 2, 175-220 (1999).

Summary: We give algorithms to find the following simply described approximation to a given matrix. Given an $m \times n$ matrix $\mathbf{A}$ with entries between say -1 and 1, and an error parameter ε between 0 and 1, we find a matrix $\mathbf{D}$ (implicitly) which is the sum of $O(1/\varepsilon^2)$ simple rank 1 matrices so that the sum of entries of any submatrix (among the 2^{m+n}) of $(\mathbf{A}-\mathbf{D})$ is at most εmn in absolute value. Our algorithm takes time dependent only on ε and the allowed probability of failure (not on m, n).

We draw on two lines of research to develop the algorithms: one is bulit around the fundamental regularity lemma of Szemerédi in graph theory and the constructive version of *N. Alon, R. A. Duke, H. Leffman, V. Rödl* and *R. Yuster* [J. Algorithms 16, 80-109 (1994; Zbl 0794.05119)]. The second one is from the papers of *Arora, Karger* and *Karpinski* [Proc. 27th Ann. ACM Symp. on Theory of Computing, 284-293 (1995)], *W. Fernandez de la Vega* [Random Struct. Algorithms 8, 187-198 (1996; Zbl 0848.90120)] and most directly *Goldwasser, Goldreich* and *Ron* [Proc. 37th Ann. IEEE Symp. on Fund. Comp. Sci. 339-348 (1996)] who develop approximation algorithms for a set of graph problems, typical of which is the maximum cut problem.

From our matrix approximation, the above graph algorithms and the regularity lemma and several other results follow in a simple way.

We generalize our approximations to multi-dimensional arrays and from that derive approximation algorithms for all dense max-SNP problems.

MSC:

68Q25 Analysis of algorithms and problem complexity
68R05 Combinatorics in computer science

Keywords:

approximation to matrices

Full Text: DOI